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Adaptive Phase-Only Algorithms for Optimal Planar Antenna Arrays

by

J C Mason and Anne E Daman

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The positioning of nulls in an antenna array field pattern is essential to the performance of the antenna, in being capable of blocking interference. The null placement must be achieved in such a way that the field pattern in other directions is not adversely affected.

One of the most efficient methods of null placement is by perturbing only the phases of the array elements. Here, we present two approaches to the placement of nulls by phase perturbation. The first is a least squares method based on exact or approximate null placement, applicable to one-dimensional arrays and extendable to two-dimensional arrays, developed for real quiescent patterns which apparently allows polygonal arrays (in this study, octagonal arrays) to be considered. The second is a minimax method in one or two dimensions based on null placement, which readily permits the omission of failed elements and which involves only the perturbation of selected element phases or amplitudes.

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We believe that these two approaches can provide between them a versatile choice. The least squares method is extremely efficient, the number of parameters being simply the number of null constraints, but all antenna weights need to be perturbed. The minimax method is much more expensive, since all antenna weights are parameters, but often only a small number of weights need to be changed. Both approaches are fairly robust. The least squares approach apparently permits polygonal arrays to be adopted, and probably more general geometries and configurations of failed elements might be adopted. The minimax approach is based on a very general optimisation algorithm and therefore in principle permits rather wide ranging specifications of constraints to be imposed by the user.

Both approaches are still under development; the theory is incomplete and algorithms have not been fully tested.

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JAMES G.R. HANSEN, Lt. Col, USAF
Chief, Structures/Struc Materials

ROBERT C. WINN, Lt. Col, USAF
Chief Scientist

ADAPTIVE PHASE-ONLY ALGORITHMS FOR OPTIMAL PLANAR ANTENNA ARRAYS

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Investigators: Professor J C Mason, Professor of Computational Mathematics
and A E Daman, Research Fellow,
Computational Mathematics and Software Group
Royal Military College of Science
Shrivenham, Swindon, Wilts
England.

CONTENTS

Note:

Sections 1-3 describe work carried out by Professor Mason and Dr Daman with financial support from the above grant. Section 4 reports on relevant work for AFOSR information, which was carried out by Professor Mason and Mr S J Wilde without AFOSR financial support.

1. INTRODUCTION

The positioning of nulls in an antenna array field pattern is essential to the performance of the antenna, in being capable of blocking interference. The null placement must be achieved in such a way that the field pattern in other directions is not adversely affected.

One of the most efficient methods of null placement is by perturbing only the phases of the array elements. ~~Here, we~~ presents two approaches to the placement of nulls by phase perturbation. The first is a least squares method based on exact or approximate null placement, applicable to one-dimensional arrays and extendable to two-dimensional arrays, developed for real quiescent patterns which apparently allows polygonal arrays (in this study, octagonal arrays) to be considered. The second is a minimax method in one or two dimensions based on null placement, which readily permits the omission of failed elements and which involves only the perturbation of selected element phases or amplitudes.

We believe that these two approaches can provide between them a versatile choice. The least squares method is extremely efficient, the number of parameters being simply the number of null constraints, but all antenna weights need to be perturbed. The minimax method is much more expensive, since all antenna weights are parameters, but often only a small number of weights need to be changed. Both approaches are fairly robust. The least squares approach apparently permits polygonal arrays to be adopted, and probably more general geometries and configurations of failed elements might be adopted. The minimax approach is based on a very general optimisation algorithm and therefore in principle permits rather wide ranging specifications of constraints to be imposed by the user.

Both approaches are still under development; the theory is incomplete and algorithms have not been fully tested. In particular no a priori guarantee can be given that the least squares technique will work for any given polygonal array, and at present it cannot be guaranteed that the minimax algorithm will always converge or that it will lead to a minimal number of phase changes. Since the current AFOSR research grant is not to continue for a second year, as originally planned, it will not be possible to complete this work under AFOSR support during the coming year.

2. PHASE-ONLY NULLING BY LEAST SQUARES METHODS IN ONE DIMENSION

We consider a linear array of N isotropic elements as shown in figure 1. The field pattern is given by,

$$p_0(u) = \sum_{n=1}^{N} a_n e^{id_n u}.$$

The weight a_n is the complex excitation of the n^{th} element. The phase reference is taken to be the centre of the array, hence the weights d_n are given by,

$$d_n = \frac{(N-1)}{2} - (n-1) = -d_{N-n+1}, \quad n=1,..N$$

and

$$u = \frac{2\pi d}{\lambda} \sin\theta,$$

where λ = wavelength,

d = interelement spacing,

θ = angle subtended with the normal to the array.

The interelement spacing is taken to be half the wavelength throughout.

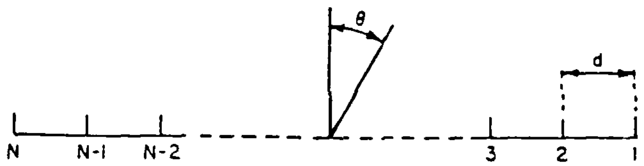

Figure 1 Geometry of the Array.

We require a set, ϕ_n, n=1,..N of phase perturbations which impose nulls in required locations whilst retaining the characteristics of the quiescent pattern. Let u_k, k=1,..K, be the directions in which nulls are required, then to obtain ϕ_n, n=1..N, we minimize the integral of the square of the discrepancy between the perturbed pattern and the original pattern, which is readily shown to equal

$$F = \sum_{n=1}^{N} c_n a_n (e^{i\phi_n} - 1)^2, \qquad (2.1)$$

(c_n are assumed to be real and positive). To impose the nulls in the required locations the phase perturbations must satisfy the following constraints,

$$\sum_{n=1}^{N} a_n e^{i\phi_n} e^{id_n u_k} = 0, \quad k=1,..K. \qquad (2.2)$$

The problem is nonlinear in general and cannot be solved analytically, but numerical solutions can be obtained using nonlinear programming.

We consider the case when the quiescent pattern is real, and hence the element weights are conjugate symmetric,

$$a_{N-n+1} = a_n^*, \; n=1,..N;$$

in this case it can be shown (Shore [1984a]) that the required phase perturbations are odd-symmetric,

$$\phi_{N-n+1} = -\phi_n. \qquad (2.3)$$

Writing the coefficients in the form

$$a_n = |a_n| e^{-id_n u_s} ,$$

then due to the odd symmetry of the pertubations ϕ_n, and the coefficients d_n, $n=1,..N$, the constraints given in equation (2.2) can be written,

$$C_k = \sum_{n=1}^{N} |a_n| \cos\{\phi_n + d_n(u_k - u_s)\}, \quad k=1,..K. \tag{2.4}$$

The objective function can be rewritten in the form,

$$F = \sum_{n=1}^{N} c_n |a_n|^2 (1-\cos\phi_n). \tag{2.5}$$

Given the objective function F, equation (2.5) and the constraints, equation (2.4), we define the Lagrangian L as follows,

$$L = F - \sum_{k=1}^{K} \lambda_k C_k, \tag{2.6}$$

where the coefficients λ_k, $k=1,..K$ are (real) Lagrangian multipliers. A necessary condition for the perturbations ϕ_n to locally minimize F subject to the constraints $C_k=0$, is that the partial derivatives of L with respect to the ϕ_n be zero. Hence a necessary condition for there to be a minimum is that

$$\frac{\partial F}{\partial \phi_n} - \sum_{k=1}^{K} \lambda_k \frac{\partial C_k}{\partial \phi_n} = 0, \quad n=1,..N. \tag{2.7}$$

Differentiating F and C_k with respect to ϕ_p, say, gives,

$$\frac{\partial F}{\partial \phi_p} = 2c_p |a_p|^2 \sin\phi_p, \quad p=1,..N \tag{2.8}$$

and

$$\frac{\partial C_k}{\partial \phi_p} = -|a_p| \sin[\phi_p + d_p(u_k - u_s)] \qquad (2.9)$$

$$p=1,..N,$$
$$k=1,..K.$$

Substituting equations (2.8) and (2.9) into (2.7) gives,

$$2c_p|a_p|^2 \sin\phi_p = - \sum_{k=1}^{K} \lambda_k |a_p| \sin[\phi_p + d_p(u_k - u_s)]$$

A little algebraic manipulation gives ϕ_p in the form

$$\tan \phi_p = \frac{- \sum_{k=1}^{K} \lambda_k \sin[d_p(u_k-u_s)]}{2c_n|a_n| + \sum_{k=1}^{K} \lambda_k \cos[d_p(u_k-u_s)]} \qquad (2.10)$$

Comparing this form for the phase perturbations ϕ_p with that given by Shore [1983], it is clear that the coefficients which Shore refers to as the 'beam coefficients' are, in this case, the negative Lagrangian multipliers. (In Shore because of the slightly different form given for the ϕ_n, n=1,..N, the coefficients are actually the negative Lagrangian multipliers divided by two.) This is the case only when the pattern is real, and hence we can take advantage of the symmetry in the coefficients ensuring the constraints are real.

The unknowns in the minimization problem are λ_k, k=1,..K , the problem size is dependent upon the number of constraints K, rather than the number of array elements N. This is clearly advantageous as the number of elements is generally much larger than the number of constraints required.

At present the problem is solved by using a NAG routine (E04VDF) which employs a sequential quadratic programming algorithm (SQP) (see Gill, Murray and Wright [1981]). The routine requires an initial estimate for the solution and routines to evaluate the objective function, constraints and their derivatives with respect to the coefficients.

The results given in table 1 are for a problem in which there are 41 antenna elements, $c_n = |a_n| = 1$, $n=1..N$, and the quiescent beam direction $u_s = 0$. The null is located at $\theta_k = 0.43633$ (rads) and the tolerence levels required by the routine were set to 1e-10. This example is taken from Shore [1983], and as stated previously the 'beam coefficients' in the formulation presented here are twice those of Shore and opposite in sign, but the resulting weight perturbations are the same.

2.1 Linearization

If the phase perturbations are assumed small, then we can employ the following approximations,

$$\tan(\phi) \approx \phi,$$

and

$$e^{i\phi_n} \approx 1 + i\phi_n.$$

The weight perturbations $w_n = a_n(e^{i\phi_n} - 1)$ can then be approximated by

$$w_n = ia_n\phi_n \tag{2.11}$$

This form for the weights allows us to rewrite the cancellation pattern, in terms of the new coefficients, as the sum of two beams

$$\Delta p(u) = \tfrac{1}{2}\sum_{k=1}^{K} \lambda_k \sum_{n=1}^{N} \frac{1}{c_n'}\left[e^{id_n(u-[2u_s-u_k])} - e^{id_n(u-u_k)}\right] \qquad (2.12)$$

where,

$$c_n' = 2c_n - \frac{1}{|a_n|} \sum_{k=1}^{K} \lambda_k \cos[d_n(u_k-u_s)]$$

One beam is in the direction of the required null, which cancels the original pattern, and the second is in the symmetrical location with respect to the main beam; which leads to an enhancement of the pattern at this point. Pattern enhancement occurs in the example described above and is illustrated in figure 2, the null at $u_k = 0.43633$ (rads) is reflected by pattern enhancement at $u = -0.43633$ (rads).

It should be noted that if we can make the approximations for ϕ_n and the weights as shown above, and if the coefficients λ_k are small relative to 2, with $c_n = |a_n| = 1$, then by neglecting any contributions from other cancellation beams at $u=u_k$, we can obtain an estimate for the coefficients as follows; the cancellation pattern can be approximated by

$$\Delta p(u_k) = -\frac{N}{4}\lambda_k, \quad k=1,..K$$

and since by definition we have

$$\Delta p(u_k) = -p_0(u_k)$$

then the coefficients can be approximated by

$$\lambda_k = \frac{4p0(u_k)}{N}. \qquad (2.13)$$

Table 2a shows the results when nulls are placed at θ = 0.59556, 0.65575, 0.71887 (rad), the approximate coefficients in this case are -0.12646, 0.11923, and -0.11349 respectively, which are clearly of the same order as the calculated values. However, table 2b illustrates the effect of imposing beams close together. As locations of nulls appr ch each other they have a greater influence on each other and the coefficients are no longer objectively independent. This is also true when imposing additional nulls close to locations symmetrical to the original nulls.

2.2. Symmetrical Nulls

It is not possible to synthesize nulls at symmetrical locations using the linearized form and approximation to the weights. (A proof is included in appendix 1 for completeness). When placing symmetrical nulls no assumption can be made about the size of the phase perturbations. Shore [1984b] considers the problem of symmetrical nulls, and concludes that it is always possible to achieve symmetrical nulls in a linear array pattern, but it may result in interference patterns. We expand these ideas here and show that in some cases it is not possible to place symmetrical nulls using this method.

If we consider the constraints and the ϕ_n for symmetrical null locations $u_k = \pm u^*$, assuming $u_s = 0$, we have from equations (2.4) and (2.10);

$$C_1 = \sum_{n=1}^{N} \cos(\phi_n + d_n u^*) = 0,$$

$$C_2 = \sum_{n=1}^{N} \cos(\phi_n - d_n u^*) = 0, \tag{2.14}$$

and

$$\tan\phi_n = \frac{\sin(d_n u^*)(\lambda_1 - \lambda_2)}{2c_n|a_n| + \cos(d_n u^*)(\lambda_1 + \lambda_2)} . \tag{2.15}$$

Using equations (2.14) and (2.15) we would like to find the conditions which specify whether there are an infinite number of solutions, no solutions or a unique solution. Shore [1984b] states that, provided that the phase perturbations are not restricted to be small, nulls can be imposed at locations symmetrical about the main beam; this is not always the case and can easily be proven. Taking the case when $c_n = |a_n| = 1$ with $\theta = \pi/6$ (rads), $u^* = \pi\sin\theta = \pi/2$. From equation (2.15) we have

$$\phi_n = \tan^{-1}\left[\frac{\sin(d_n\pi/2)(\lambda_1 - \lambda_2)}{2 + \cos(d_n/2)(\lambda_1 + \lambda_2)}\right] \quad n=1,..N. \tag{2.16}$$

When d_n is even, $\sin(d_n\pi/2) = 0$, and therefore $\phi_n = t\pi$, t is an integer; when d_n is odd, $\cos(d_n\pi/2) = 0$, and therefore $\phi_n = \pm \tan^{-1}(\lambda_1 - \lambda_2)/2$.

The terms in the summation of the constraints given by equation (2.15) are:

(i) for even d_n, $\cos(\phi_n \pm d_n\pi/2) = \cos(t_1\pi) = \pm 1$,

(ii) for odd d_n, $\cos(\phi_n \pm d_n\pi/2) = \cos(T \pm \pi/2)$,

where $T = \tan^{-1}(\lambda_1 - \lambda_2)/2$. The summation of the terms results in the pair of equations

$$\frac{N-1}{2}\cos(T\pm\frac{T}{2}) + k = 0 \quad ,$$

where k is a non-zero integer. This leads to the equations,

$$\sin T = \pm \frac{2k}{N-1} \quad ,$$

which indicates that the constraints are two parallel lines and thus can never both be satisfied. In this case there is no solution to the constrained minimization problem. (Here it is assumed that (N-1)/2 is odd, however the result is of the same form if (N-1)/2 is even).

In general there is a solution to the constrained minimization problem and the amount of interference which occurs is dependent on the position of the null relative to a null in the quiescent pattern. Figure 3 illustrates this relationship by placing nulls at intervals between quiescent nulls of a 41 element array.

There appears to be a relationship between the optimal coefficients and the unperturbed pattern value but we have been unable to establish that relationship at present.

2.3. Increasing null width by Higher Order Constraints

Since the placing of nulls at locations close together results in cancellation beams interference, as illustrated in the previous section, it may prove wise to employ alternative methods for increasing the width of a null.

Consider the constraints

$$\frac{d^{\nu}}{du^{\nu}} p_a(u_k) = 0, \quad \begin{matrix} k=1,..K \\ \nu=0,..M \end{matrix} \qquad (2.17)$$

where u_k denotes the location of the K interference directions. Previously we have considered $\nu=0$, however, by including higher order derivatives, the null width is broadened. This was illustrated for null synthesis with phase end amplitude perturbations by Steyskal [1982]. Here, we investigate the use of higher order constraints in the context of phase-only nulling.

Clearly now the total number of constraints is P=K(M+1), and they are given by

$$C_p = \sum_{n=1}^{N} (d_n)^\nu \, |a_n| \cos[\phi_n + d_n (u_k-u_s)] \quad \text{for even } \nu, \tag{2.18}$$

and

$$C_p = \sum_{n=1}^{N} (d_n)^\nu \, |a_n| \sin[\phi_n + d_n (u_k-u_s)] \quad \text{for odd } \nu. \tag{2.19}$$

The Lagrangian is now of the form

$$L = F - \sum_{p=1}^{P} \lambda_p C_p \quad ,$$

and again the condition for d minimum is

$$\frac{\partial F}{\partial \phi_n} - \sum_{p=1}^{P} \lambda_p \frac{\partial C_p}{\partial \phi_n} = 0 \tag{2.20}$$

From equations (2.18) and (2.19) the derivative of the constraints are given by:-

$$\frac{\partial C_p}{\partial \phi_n} = - |a_n| \, (d_n)^\nu \sin[\phi_n + d_n(u_k-u_s)] \quad \text{for even } \nu, \tag{2.21}$$

and

$$\frac{\partial C_p}{\partial \phi_n} = |a_n| (d_n)^\nu \cos[\phi_n + d_n(u_k-u_s)] \quad \text{for odd } \nu. \tag{2.22}$$

From equations (2.9), (2.20), (2.21) and (2.22) we have

$$2c_n|a_n|\sin\phi_n = -\sum_{k=1}^{K} \sum_{\substack{\nu=0 \\ \text{step } 2}}^{M} \lambda_{k\nu}(d_n)^\nu \sin[\phi_n + d_n(u_k-u_s)]$$

$$+\sum_{k=1}^{K} \sum_{\substack{\nu=1 \\ \text{step } 2}}^{M} \lambda_{k\nu}(d_n)^\nu \cos[\phi_n+d_n(u_k-u_s)],$$

where $k\nu = (k-1)(M+1)+(\nu+1)$. After a little algebraic manipulation we obtain,

$$\tan\phi_n = \frac{-\sum_{k=1}^{K}\sum_{\substack{\nu=0,2..\\ \text{step } 2}}^{M}\lambda_{k\nu}(d_n)^{\nu}\sin d_n(u_n-u_s) + \sum_{k=1}\sum_{\substack{\nu=1,3..\\ \text{step } 2}}\lambda_{k\nu}(d_n)^{\nu}\cos d_n(u_k-u_s)}{2c_n|a_n| + \sum_{k=1}^{K}\sum_{\nu=0,2..}^{M}\lambda_{k\nu}(d_n)^{\nu}\cos d_n(u_k-u_s) - \sum_{k=1}^{K}\sum_{\nu=1,3..}^{M}\lambda_{k\nu}(d_n)^{\nu}\sin d_n(u_k-u_s)} \tag{2.23}$$

If $\nu = 0$ then (2.23) reduces to equation (2.7).

The problem can be solved in the same manner as previously except now there are $K(M+1)$ variables instead of K. The method used to solve the constrained minimization problem requires derivatives of both the objective function and the constraints, the equations for these are given in appendix II.

2.3.1 Numerical Results

The effect of including higher order constraints is illustrated in figures 4, the corresponding coefficients are given in table 3. Figure 4(a) illustrates the quiescent pattern, with 31 elements and $u_s=0$ and $a_n=1$ $n=1,\ldots N$. Figure 4(b) illustrates the pattern with a zero order null located at $u_k=0.3$, the null is indicated by the vertical line. Figure 4(c) and (d) illustrate clearly how the null is broadened by the addition of higher order nulls, with a first order and second order null illustrated respectively. It is evident from the coefficients in Table 3 that in this example the original beam coefficient does not vary greatly with the addition of higher order constraints. However the whole pattern is affected by the higher order constraint, resulting in some amount of pattern enhancement on the half range symmetrical to the null location.

3. PHASE ONLY NULLING BY LEAST SQUARES METHODS IN TWO DIMENSIONS

3.1 Notation

For a two-dimensional planar array the phase equation is of the form,

$$\theta_{k,\ell} = \frac{2\pi}{\lambda} d_x \, wx_k \, u + \frac{2\pi}{\lambda} d_y \, wy_\ell \, v \tag{3.1}$$

$$k=1..N_x, \quad \ell=1,..N_y,$$

and the resulting field pattern is given by

$$p = \sum_{n=1}^{N_x} \sum_{m=1}^{N_y} a_{n,m} \, e^{i\theta_{n,m}} \tag{3.2}$$

Here

N_x	= number of elements in the x-direction
N_y	= number of elements in the y-direction
wx_k	= $k-(n_x+1)/2$, $k=1,..N_x$
wy_ℓ	= $\ell-(N_y+1)/2$, $\ell=1,..N_y$
k,ℓ or m,n	= the reference coordinates in the x,y plane
d_x, d_y	= element spacing in the x,y directions respectively
$a_{n,m}$	= complex excitation of the n,m^{th} element
u,v	= positions in the coordinate system shown below.

and $u = \sin\psi \cos\phi$, $v = \sin\psi \sin\phi$,

where the angles ψ and ϕ are as illustrated in figure 5.

We assume that the element spacings d_x and d_y are half wavelength. We also assume that the quiescent pattern is real, and in that case the coefficients $a_{n,m}$ exhibit a symmetry about the centre element of the array. We shall return to this point later.

3.2 A Two-Dimensional Planar Array

We require a phase change in the element coefficients which will result in the placement of a null in a given direction whilst replicating the remaining field pattern.

Given a set of points $\{U_k, V_k\}$ k=1,..K at which nulls are to be placed, then we require

$$p(U_k,\ V_k) = 0 \qquad k=1,..K \tag{3.3}$$

Let us consider the geometry of the array. If, given a grid of elements, the main beam is assumed to be central to the array, (that is the weights wx_n and wy_m are as defined above), then the phase equation at each element is as illustrated in figure 6. (The example here is for a 5x5 grid, but clearly the basic pattern is the same for a general grid).

If the elements are numbered in the order illustrated in figure 6, it can easily be shown that in order for the field pattern to be real, the complex coefficients $a_{m,n}$ must be conjugate symmetric about the central element (in this case element number 13).

Ordering the coefficients in this manner, as a vector, we have

$$a_{m,n} \rightarrow \underline{a}(j) \qquad \begin{array}{c} j = m(n-1) + m \\ j=1,..N \\ N = N_x \times N_y . \end{array} \tag{3.4}$$

We can now consider the field pattern, given in (3.2) in a vector form,

$$p = \sum_{j=1}^{N} a_j \, e^{i\theta_j}, \tag{3.5}$$

and in effect we now have a one-dimensional problem.

If the unknown phase perturbations required to place a null (or set of nulls) is denoted by $\{\phi_j\}$ $j=1,..N$, then from equation (3.3), we have the null constraints,

$$\sum_{j=1}^{N} a_j \, e^{i\phi_j} \, e^{i\theta_k} = 0 \qquad k=1,..K \tag{3.6}$$

To ensure that the perturbed pattern replicates the quiescent pattern everywhere but at the null location, we minimize the sum of squares *of the absolute element perturbations*

$$F = \sum_{j=1}^{N} c_j \, |a_j(e^{i\pi_j} - 1)|^2 \tag{3.7}$$

where c_j $j=1,..N$ are positive weights.

Owing to the conjugate symmetry of the coefficients, this can be written in the form,

$$F = 2 \sum_{j=1}^{n} c_j \, |a_j|^2 \quad (1-\cos\theta_j). \tag{3.8}$$

The null constraints given in (3.6) can also be re-written as

$$0 = C_k = \sum_{j=1}^{N} a_j \, \ell^{i\theta_k} \, \ell^{i\theta_j}$$

$$= \sum_{j=1}^{n} |a_j| \cos [\phi_j + \theta_k] \qquad k=1,..K \tag{3.9}$$

As in the one-dimensional problem, we form the Lagrangian

$$L = F - \sum_{p=1}^{K} \lambda_p C_p, \tag{3.10}$$

and the problem has a 'beam space' solution just as for the one-dimensional problem. From (3.8), (3.9) and (3.10) we can obtain the relationship

$$\tan (\phi_p) = \frac{- \sum_{k=1}^{K} \lambda_k \sin [d_p(U_k-U_s)]}{2 C_n|a_n| + \sum_{k=1}^{K} \lambda_u \cos [d_p (U_k-U_s)]} \tag{3.11}$$

Details of the algebra behind this relationship are as discussed in §2.

At this stage it is a simple matter to show that all the results for the one-dimensional case hold in two dimensions.

Clearly the number of variables in the optimization problem, namely that of minimizing (3.8) whilst satisfying the constraints given in (3.9), is reduced from the number of elements N_x xN_y to the number of constraints K, by utilizing (3.11).

Just as for the one-dimensional array, in some cases there is an enhancement of the pattern in a direction symmetric to the location of the null. Examples of this phenomenon will be illustrated in the numerical results given below.

3.3 Polygonal arrays

Billam (1985) poses the question of the suitability of phase only nulling for an octagonal array of elements.

It is possible to embed an octagonal array of elements in a rectangular grid; as shown in figure 7. All the elements of the rectangular array which lie outside the octagon are clearly in symmetric positions about the central element. By putting the initial weights of these elements to zero and ensuring that they are eliminated from further calculations, it is possible to simulate the problem of null placement in an octagonal antenna field pattern.

This method of embedding can in principle be extended to any polygonal geometry of array.

We have developed computer routines for the embedding of an octagonal array into a rectangular array, and the placement of nulls by phase perturbations to the octagonal array elements.

A listing of the embedding routine, which embeds the octagonal array into a suitable rectangle by setting the appropriate weights to zero, can be found in appendix III; it is quite self-explanatory. In the section on numerical results below, we illustrate the difference in the quiescent field for the octagonal and the rectangular arrays, show how single and multiple nulls can be achieved for each, and the effect on other areas of the pattern.

3.4 Numerical Results

For the following results, the optimization routine used to solve the constrained minimization problem was taken from the Harwell library of optimization routines (VF13). The method is based on a quadratic programming technique and is described in Powell (1982) and Chamberlain et al. (1982). Linear approximations are made to the non-linear constraints, and hence the placement of nulls at symmetric locations would not be posible using the routine. (See §2.2 above).

The routine requires the evaluation of the objective function and constraints, plus their first derivatives.

The examples below illustrate single and multiple null placement for a rectangular antenna array and an octagonal antenna array. Figures 8(a) and 8(b) illustrate the quiescent sinc pattern for both the rectangular array and the octagonal array respectively with a grid of 13x13 elements. The octagonal array, which is embedded into the 13x13 grid of elements, has 5 elements along each face, and clearly the resulting pattern is more circular in shape.

For each case we have placed a null at $u = 0.28$, $v = 0.32$, with a tolerence of 10^{-8} allowed on the constraint, and an initial estimate of the beam space coefficient taken as 0.1. Figures 9 and 10 illustrate the resulting perturbed patterns and the difference between the perturbed and the quiescent pattern for a rectangular array and an octagonal array respectively. The coefficients for the perturbed pattern are given in each case in tables 4 and 5, respectively, (only the first half of the coefficients need be given, owing to symmetry). It is clear from these tables that a null placed in the octagonal pattern results in a larger absolute beam coefficient, and this in turn results in a higher average perturbation but leads to a deeper null.

On inspecting the graphical results, it is clear from the difference patterns (i.e. the differences between perturbed and quiescent patterns) that the octagonal pattern is affected slightly more

The least-squares method applied to both rectangular and octagonal array phase-only nulling problems has always given good results in the cases considered so far.

4. NULL PLACEMENT BY MINIMAX METHODS

Given an initial far field pattern

$$P_0(u) = \sum_{n=1}^{N} a_n e^{id_n u} \tag{4.1}$$

the perturbed pattern becomes

$$p(u) = \sum_{n=1}^{N} x_n e^{id_n u} \tag{4.2}$$

Denoting the discrepancy by e(u) then

$$e(u) = R(u) + iI(u) = \sum_{n=1}^{N} (a_n - x_n) e^{id_n u} \tag{4.3}$$

For amplitude-only perturbation x_n/a_n is real (and so is a_n), and for phase only perturbations

$$x_n/a_n = e^{i\theta_n}$$

where θ_n is a real parameter.

4.1 Amplitude-only Nulling

Although expensive to implement, null placement is possible by very few perturbations to real weights. Adopting a minimax criterion, we require to find the parameters x_n $(n=1,\ldots,N)$ which minimise $\|e\|_\infty$, where

$$\|e\|_\infty = \max_{-1 \leq u \leq 1} |e(u)|$$

$$= \max_{-1 \leq u \leq 1} \{R^2(u) + I^2(u)\}^{1/2}$$

subject to the constraints

$$-\varepsilon \leq \{\mathrm{Re}\, |p(u)| \,,\ \mathrm{Im}|p(u)|\} \leq \varepsilon$$

for $u = u_i$ $\qquad i=1,\ldots,p$

(where u_i are discrete locations at which "near-null" placements are to be made).

The expression (4.4) is nonlinear in the unknown parameters x_n, but may be linearized, with a relative loss in accuracy of at most $\sqrt{2}$ by being replaced by

$$\|e\|_\infty^* = \max_{-1 \leq u \leq 1} \left[\max\{\, |R(u)|,\ |I(u)|\}\right]$$

[See Barrodale, Delves and Mason (1978)]

The problem is now an overdetermined linear programming problem and can be solved by a standard routine such as that of Roberts and Barrodale (1980).

Figures 13 to 15 show 3 model 41 element quiescent patterns, based on Sinc, 20 dB Taylor weighted and 30 dB Taylor weighted patterns, respectively.

In Figures 16 and 17 we have given examples of null placement for the 41 element sinc pattern. Note that the weights are symmetric, thus reducing the dimensions of the problem by half. Also, because the perturbations are amplitude only, the pattern is symmetric about boresight. In Figure 16, to achieve a null interval [.7, .73], only

3 pairs of weights are changed from their quiescent values of 1. In Figure 17 to achieve a null interval [.07, .08] just 2 pairs of weights are changed from unity.

The technique described here (for amplitude-only changes) has already been discussed with different numerical examples, by Mason, Wilde and Opfer (1987).

4.2 Phase-only Perturbations

For phase only perturbations, the constrained pattern is

$$\sum_{n=1}^{N} a_n \, e^{(id_n u + \theta_n)} \tag{4.5}$$

and

$$E(u) = \sum_{n=1}^{N} a_n \, e^{id_n u} - \sum_{n=1}^{N} a_n e^{id_n u + \theta_n}$$

$$= \sum_{n=1}^{N} a_n \, (e^{id_n u} - e^{id_n u + \theta_n})$$

$$= R(u) + iI(u)$$

where

$$R(u) = \sum_{n=1}^{N} a_n \left[\cos(d_n u) - \cos(d_n u + \theta_n)\right] \tag{4.6}$$

$$I(u) = \sum_{n=1}^{N} a_n \left[\sin(d_n u) - \sin(d_n u + \theta_n)\right] \tag{4.7}$$

Here we minimise

$$\|e\|^* = \max_{\substack{-1 \leq u_i \leq u_a \\ u_b \leq u_i \leq 1}} \{\max(|R(u)|,\ |I(u)|)\} \qquad (4.8)$$

for a discrete set of u_i (i=1,...m) covering the range of minimisation, subject to the constraints

$$\left.\begin{aligned} -\ \varepsilon \leq R(u) \leq \varepsilon \\ \\ -\ \varepsilon \leq I(u) \leq \varepsilon \end{aligned} \qquad u^b[u_a,u_b] \right\}$$

for a discrete set of u_i i=m,m+1m,...,m+p covering the range of nulls.

The minimization is achieved by imposing inequalities

$$\left.\begin{aligned} -z \leq R(u_i) \leq z \\ -z \leq I(u_i) \leq z \end{aligned}\right\} \qquad (4.9)$$

and minimising z (in place of $\|e\|^*$).

The inequalities (4.9) give

$$\left.\begin{aligned} & -z \leq \sum_{n=1}^{N} a_n \left(\cos(d_n\, u_i) - \cos(d_n\, u_i + \theta_n)\right) \leq z \\ & \text{and} \\ & -z \leq \sum_{n=1}^{N} a_n \left(\sin(d_n\, u_i) - \sin(d_n\, u_i + \theta_n)\right) \leq z \end{aligned}\right\} \qquad (4.10)$$

which become:

$$\left.\begin{aligned}
\sum_{n=1}^{N} a_n \cos(d_n u_i) &\le \sum_{n=1}^{N} a_n \cos(d_n u_i + \theta_n) + z \\
\sum_{n=1}^{N} a_n \cos(d_n u_i) &\ge \sum_{n=1}^{N} a_n \cos(d_n u_i + \theta_n) - z \\
\sum_{n=1}^{N} a_n \sin(d_n u_i) &\le \sum_{n=1}^{N} a_n \sin(d_n u_i + \theta_n) + z \\
\sum_{n=1}^{N} a_n \sin(d_n u_i) &\ge \sum_{n=1}^{N} a_n \sin(d_n u_i + \theta_n) - z
\end{aligned}\right\} \tag{4.11}$$

Hence the problem has $2p+4m$ nonlinear constraints, $N+1$ variables $(\theta_n(n=1,\ldots N),z)$ and a linear objective function z. For its solution we have used the NAG sequential quadratic programming routine E04VDF, as discussed in §2 above.

We have tackled a wide variety of problems, some of which are shown in Figures 18-22. The tables 8-11 give numerical information which corresponds, respectively, to these 5 figures.

For a symmetrically weighted array, if we limit the phase changes to $[-\pi,\pi]$ then this results in conjugate symmetric perturbations as in the least squares approach. However, it is interesting to note that on restricting the phase perturbation to $[0,2\pi]$, we produce a sub-optimal solution but, by the nature of the optimization procedure, few of the elements undergo changes. This is illustrated clearly in Figures 21-22, for nulls .22, .24, .26, .28; here conjugate symmetric results are obtained in Fig 21 but all weights are changed, while non-symmetric results are obtained in Fig 22 with just 10 changes to the 41 weights.

4.3 Loss of elements

The loss of elements in the array can readily be counteracted by applying phase changes to the remaining weights so as to approximate the original perturbed pattern. In the algorithms of §§4.1, 4.2, we simply set the failed elements to zero and minimize with respect to the remaining elements.

This technique is illustrated successfully in Figures 23-26. In Figure 23(a) is shown a perturbed pattern with no failed element, obtained from a 20 dB Taylor weighted pattern, to achieve nulls in [0.7, 0.72]. In Figures 23(b) - 23(d), one, two and three elements have failed and the resulting pattern is successfully adjusted by the minimax algorithm.

4.4 Two-Dimensional Arrays

The techniques of §§4.2, 4.3 are equally applicable to two-dimensional arrays. However, the minimization problems can become rather large in that case, and so more efficient but algebraically complicated techniques such as that of Streit (1985) should probably be adopted for processing the linear inequalities.

5. SUMMARY OF PROGRESS

The proposed program of work for the 2-year contract was as follows:

(i) To test a constrained least squares method for adapting a planar phased array to known interference directions

(ii) To test a constrained minimax method analogous to (i)

(iii) To develop and test new algorithms for improving (i), (ii), based on novel research ideas.

(iv) To extend the method of Thompson (1976) and other related methods for designing adaptive planar arrays and to develop sound nonlinear optimization techniques for the necessary minimization procedures.

Substantial progress was made in the first year of the contract on tasks (i), (ii) and (iii) as follows:

A modified version, based on Lagrange multipliers, of Shore's beam space representation method for least squares phased array adaptation was introduced. In this implementation the Lagrange multipliers were in fact the beam space coefficients. The method was extended to planar arrays, and also successfully applied to octagonal arrays. In addition multiple nulls were shown to be readily introduced, and formulae for corresponding Lagrange multipliers obtained. Symmetry was considered, and problems of pattern enhancement at positions symmetrical to nulls were studied; it was shown that it is sometimes not possible to place symmetrical nulls in beam space-type algorithms.

We also reported on work carried out without AFOSR financial support on minimax methods for null placement in antenna patterns. We noted that algorithms, analogous to those of Mason, Wilde and Opfer (1987) for amplitude-only nulling, could be applied to the phase-only nulling problem. If phase changes were restricted to ranges $[-\pi,\pi]$ then the odd symmetry of the problem was adhered to. If phase changes were restricted to $[0,2\pi]$, then symmetry was not achieved, but very few phase changes were required in this case. It was also noted that failed elements were readily catered for in this type of approach.

The work carried out so far still requires further theoretical development and numerical testing, before we can guarantee the full efficacy of the techniques discussed. However, the numerical results produced have been consistently good, and so we see considerable promise in the ideas introduced.

6. REFERENCES

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7 APPENDICES

7.1 Appendix I

If it is assumed that the phase perturbation will be small, so that we can use the linearized form of the weights given in equation (2.11), then, for symmetrical nulls at $\pm u_k$, the constraints given by equation (2.2) are:

$$\sum_{n=1}^{N} a_n e^{i\phi_n} e^{id_n u_k} = 0$$

and (I.1)

$$\sum_{n=1}^{N} a_n e^{i\phi_n} e^{-id_n u_k} = 0.$$

Putting $e^{i\phi_n} = 1 + i\phi_n$, equations (I.1) become,

$$-i \sum_{n=1}^{N} a_n \phi_n e^{id_n u_k} = p_0(u_k).$$

and (I.2)

$$-i \sum_{n=1}^{N} a_n \phi_n e^{-id_n u_k} = p_0(u_k).$$

Here $p_0(u_k)$ is the value of the unperturbed pattern at u_k. Clearly the left hand sides of equations (I.2) are complex conjugates and due to the odd symmetry of ϕ_n and d_n the resulting equations are inconsistent. Therefore it is not possible to gain a solution using the linearized form for symmetric null placement.

7.2 Appendix II

Given,

$$F = 2\sum_{n=1}^{N} c_n|a_n|^2\cos\phi_n,$$

then the derivative is given by,

$$\frac{\partial F}{\partial\lambda_p} = 2\sum_{n=1}^{N} c_n|a_n|^2\sin\phi_n \frac{\partial\phi_n}{\partial\lambda_p}\ ,\ p=1,..P. \qquad \text{(II.1)}$$

Also,

$$\frac{\partial C_s}{\partial\lambda_p} = \sum_{n=1}^{N} \frac{\partial C_s}{\partial\phi_n}\frac{\partial\phi_n}{\partial\lambda_p} \quad \begin{array}{l} s=1,..P, \\ p=1,..P. \end{array} \qquad \text{(II.2)}$$

and $\frac{\partial C_s}{\partial\phi_n}$ is given by equations (2.21) and (2.22).

From equation (2.23) we have,

$$\phi_n = \tan^{-1}\frac{-\sum\limits_{k=1}^{K}\sum\limits_{\substack{\nu=0,2..\\ \text{step } 2}}^{M}\lambda_{k\nu}(d_n)^\nu\sin[d_n(u_n-u_s)] + \sum\limits_{k=1}^{K}\sum\limits_{\substack{\nu=1,3..\\ \text{step}}}^{M}\lambda_{k\nu}(d_n)^\nu\cos[d_n(u_k-u_s)]}{2c_n|a_n| + \sum\limits_{k=1}^{K}\sum\limits_{\nu=0,2..}^{M}\lambda_{k\nu}(d_n)^\nu\cos[d_n(u_k-u_s)] - \sum\limits_{k=1}^{K}\sum\limits_{\nu=1,3..}^{M}\lambda_{k\nu}(d_n)^\nu\sin[d_n(u_k-u_s)]} \qquad \text{(II.3)}$$

Putting

$$y_n = \tan\phi_n \qquad \text{(II.4)}$$

then

$$\frac{\partial\phi_n}{\partial\lambda_p} = \frac{d\phi_n}{dy_n}\frac{\partial y_n}{\partial\lambda_p}\ , \qquad \text{(II.5)}$$

and

$$\frac{\partial\phi_n}{\partial y_n} = \cos^2\phi_n. \qquad \text{(II.6)}$$

Denoting,

$$v_n = 2c_n|a_n| + \sum_{k\ \text{even}}\sum_{\nu}\lambda_{k\nu}(d_n)^\nu\cos[d_n(u_k-u_s)] - \sum_{k\ \text{odd}}\sum_{\nu}\lambda_{k\nu}(d_n)^\nu\sin[d_n(u_k-u_s)]$$

and

$$t_n = -\sum_k \sum_{\text{even } \nu} \lambda_{k\nu}(d_n)^\nu \sin[d_n(u_k-u_s)] + \sum_k \sum_{\text{odd } \nu} \lambda_{k\nu}(d_n)^\nu \cos[d_n(u_k-u_s)],$$

then

$$\frac{\partial y_n}{\partial \lambda_p} = -\frac{(d_n)^\nu}{v_n}\sin[d_n(u_k-u_s)] + (d_n)^\nu \cos[d_n(u_k-u_s)]\frac{t_n}{v_n^2} \quad \text{for even } \nu, \quad \textbf{(II.7)}$$

and

$$\frac{\partial y_n}{\partial \lambda_p} = \frac{(d_n)^\nu}{v_n}\cos[d_n(u_k-u_s)] - (d_n)^\nu \sin[d_n(u_k-u_s)]\frac{t_n}{v_n^2} \quad \text{for odd } \nu \quad \textbf{(II.8)}$$

Equations (II.7) and (II.8) together with (II.6) (2.21), (2.22), (II.2) and (II.1) give all the required derivatives for the objective function and constraints.

7.3 Appendix III Listing of Routine for Setting non-Octagon weights to Zero.

```
      SUBROUTINE WEIGHT(A, C, NELEM, NADD, NSOCT, MAXELT, POLY)
C
C     IN THIS ROUTINE THE WEIGHTS A AND C ARE SET ACCORDING TO
C     THE TYPE OF ARRAY, RECTANGULAR (R) OR OCTAGONAL (O).
C     NELEM IS TOTAL NUMBER OF ELEMENTS IN RECTANGULAR ARRAY.
C     NSOCT IS NUMBER OF ELEMENTS ALONG EDGE OF OCTAGONAL ARRAY.

C     NADD IS NUMBER OF ELEMENTS ADDED TO EACH SIDE OF OCTAGONAL

C          EDGE TO EMBED IT INTO RECTANGLE.
C
      IMPLICIT DOUBLE PRECISION (A-H, P-Z)
      CHARACTER*1 POLY
C
      DIMENSION A(MAXELT),C(MAXELT)
C
C    FIRST SET ALL WEIGHTS TO 1, FOR SINC PATTERN
C
      DO 10 JELEM = 1, NELEM
        C(JELEM) = 1.0d0
        A(JELEM) = 1.0d0
   10 CONTINUE
C
      IF (LIT.EQ.'R'.OR.LIT.EQ.'r') RETURN
C
C   NOW PLACE ZEROS FOR OCTAGONAL
C
      DO 20 JELEM = 1, NADD
        A(JELEM) = 0.0D0
        A(NELEM-JELEM+1) = 0.0D0
   20 CONTINUE
C
      NSTART = NSOCT +NADD
      NUMAD = NSOCT
      NZERO = 2*NADD -1
C
      DO 25 INUM = 1, NADD
        DO 23 JELEM = 1, NZERO
          NTJ = NSTART + JELEM
          A(NTJ) = 0.0D0
          A(NELEM-NTJ+1) = 0.0D0
   23   CONTINUE
C
        NUMAD = NUMAD+2
        NSTART = NTJ+NUMAD
        NZERO = NZERO -2
   25 CONTINUE
C
      RETURN
      END
```

List of Figures

Figure 2. Enhancement of pattern at Symmetric Location to Null: Uk=0.43633

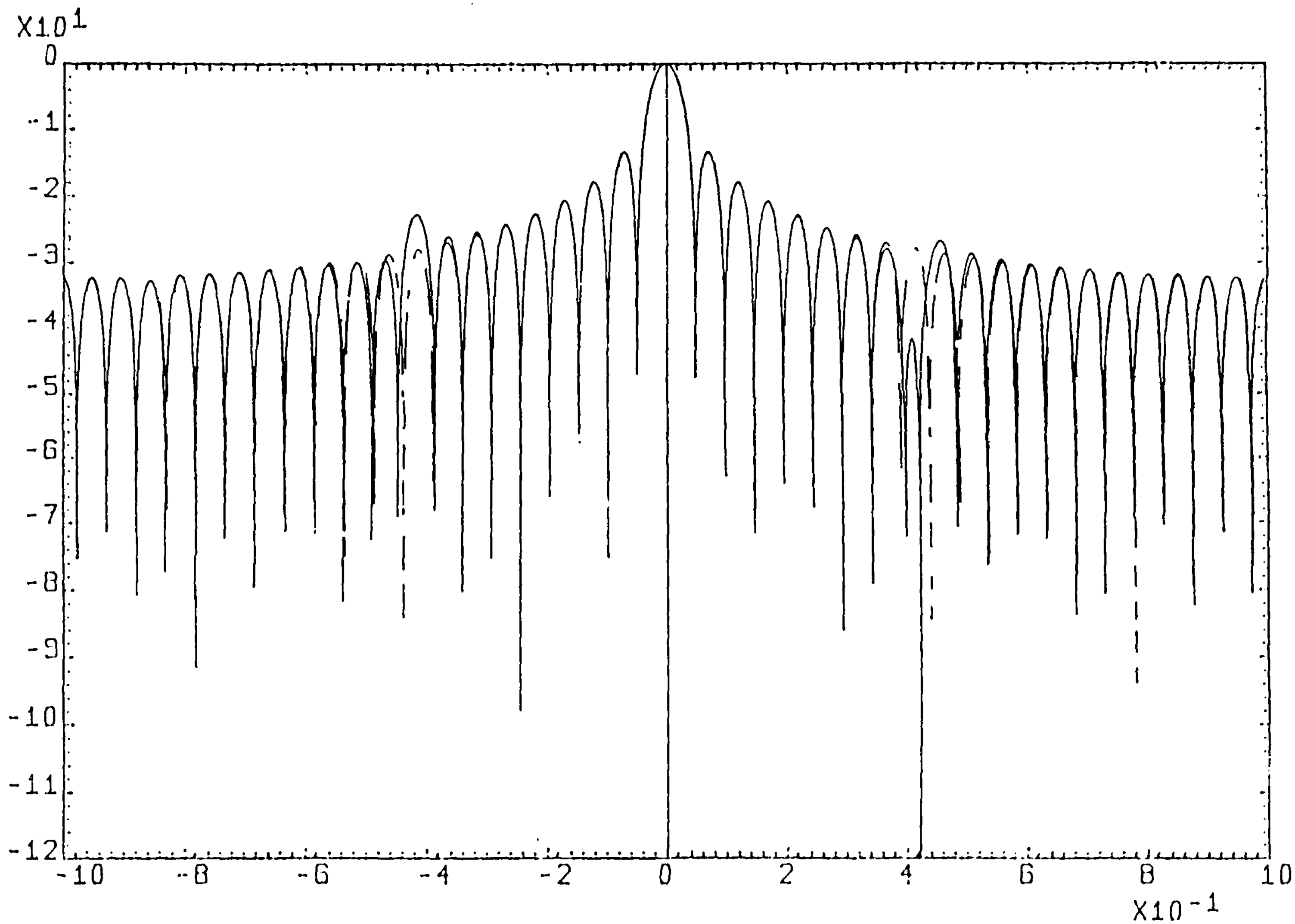
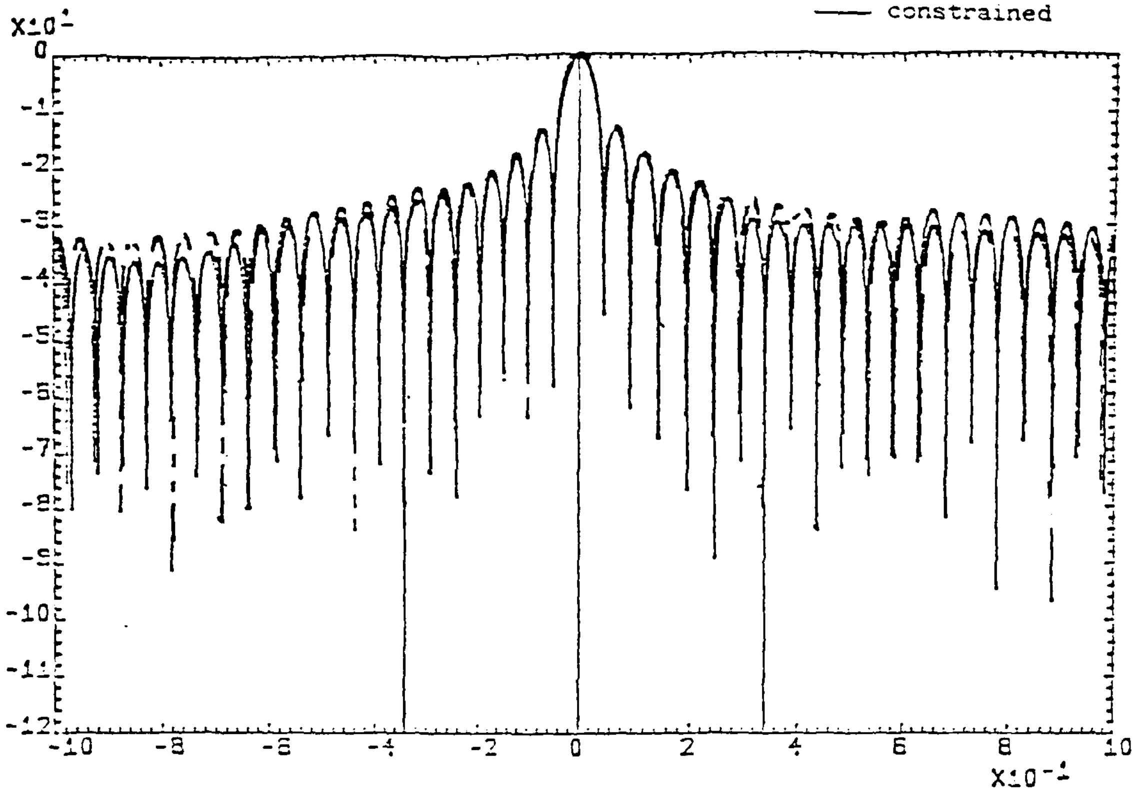
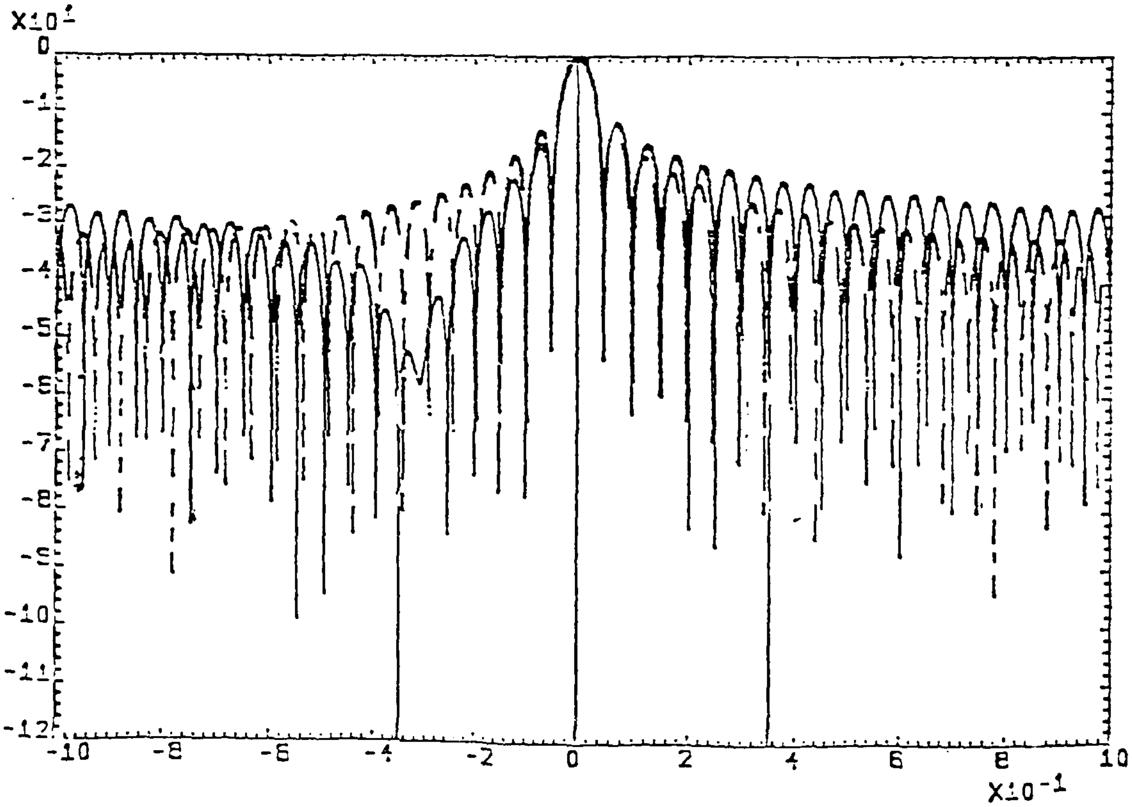


Figure 3 : Placement of Symmetrical Nulls at locations between Nulls of Quiescent Pattern.

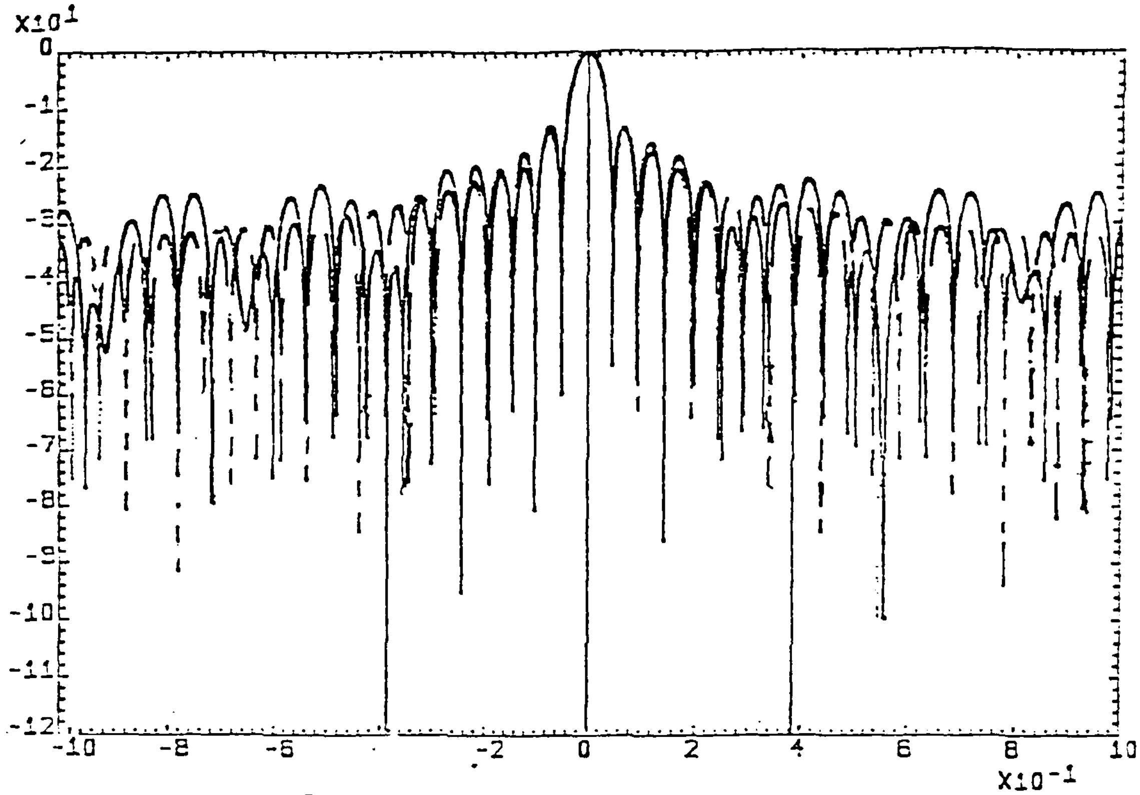
Key: ---- quiescent
—— constrained



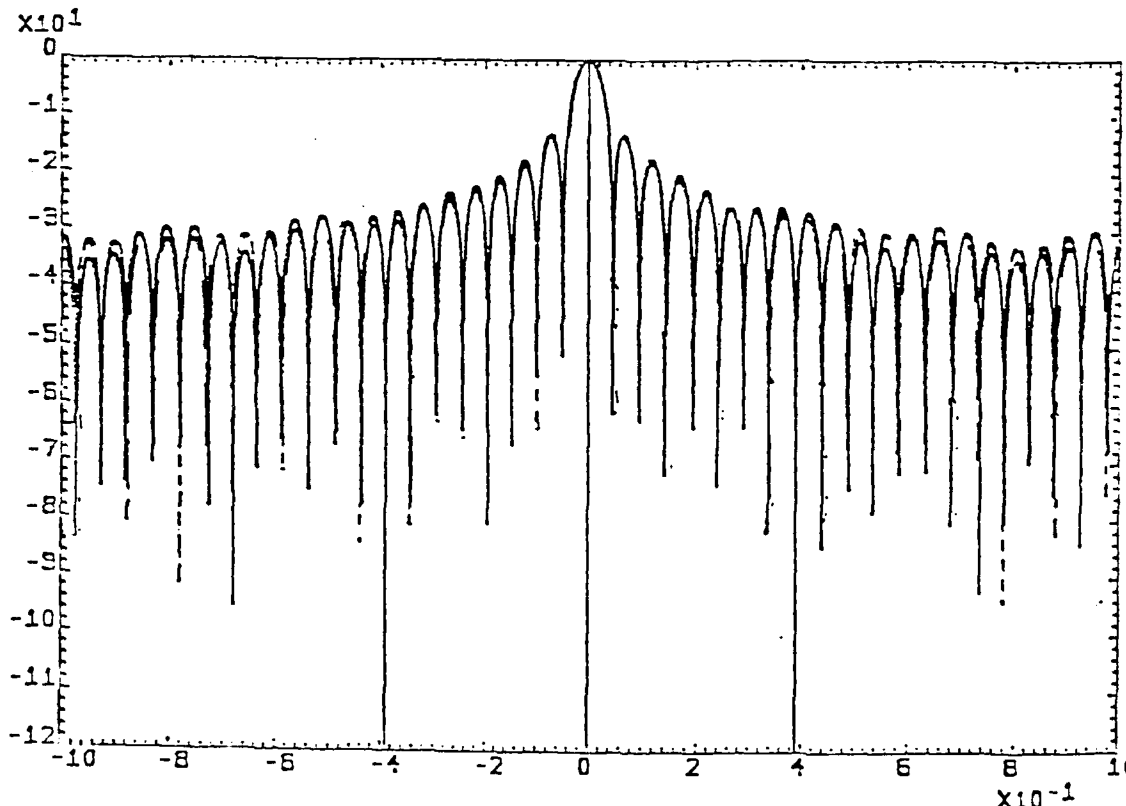
(a) Uk = ± 0.341



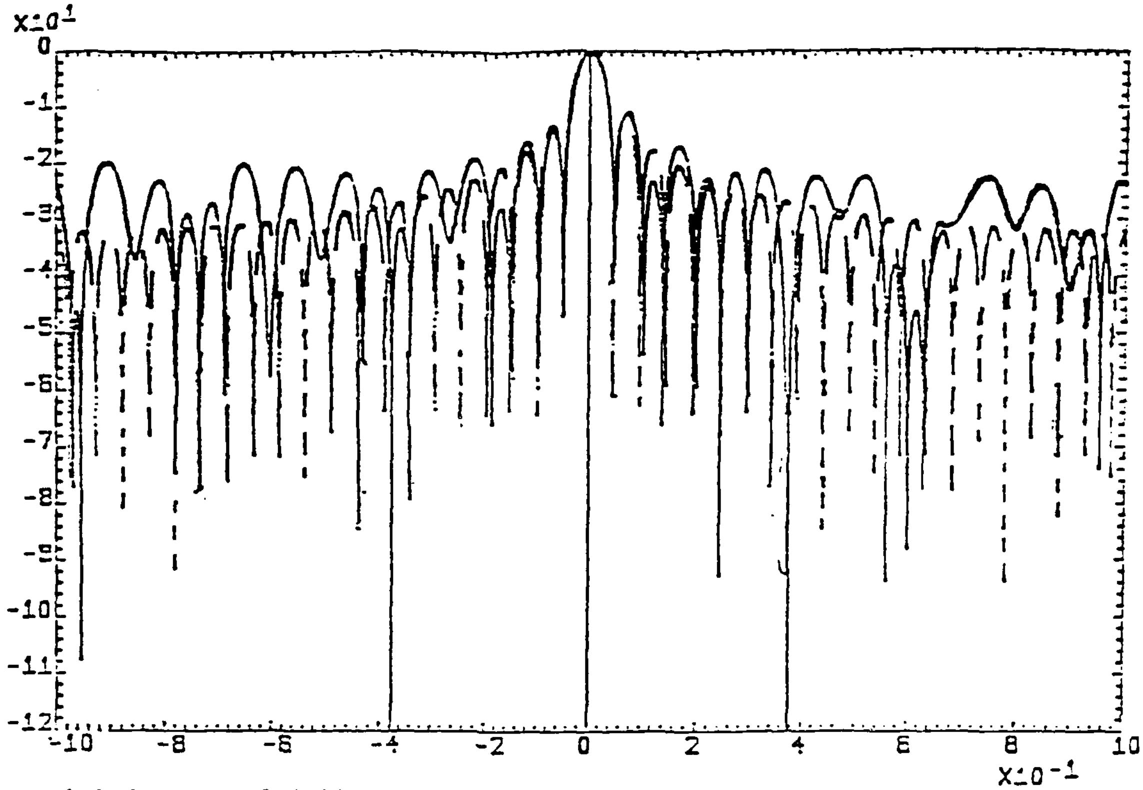
(b) Uk = ± 0.351



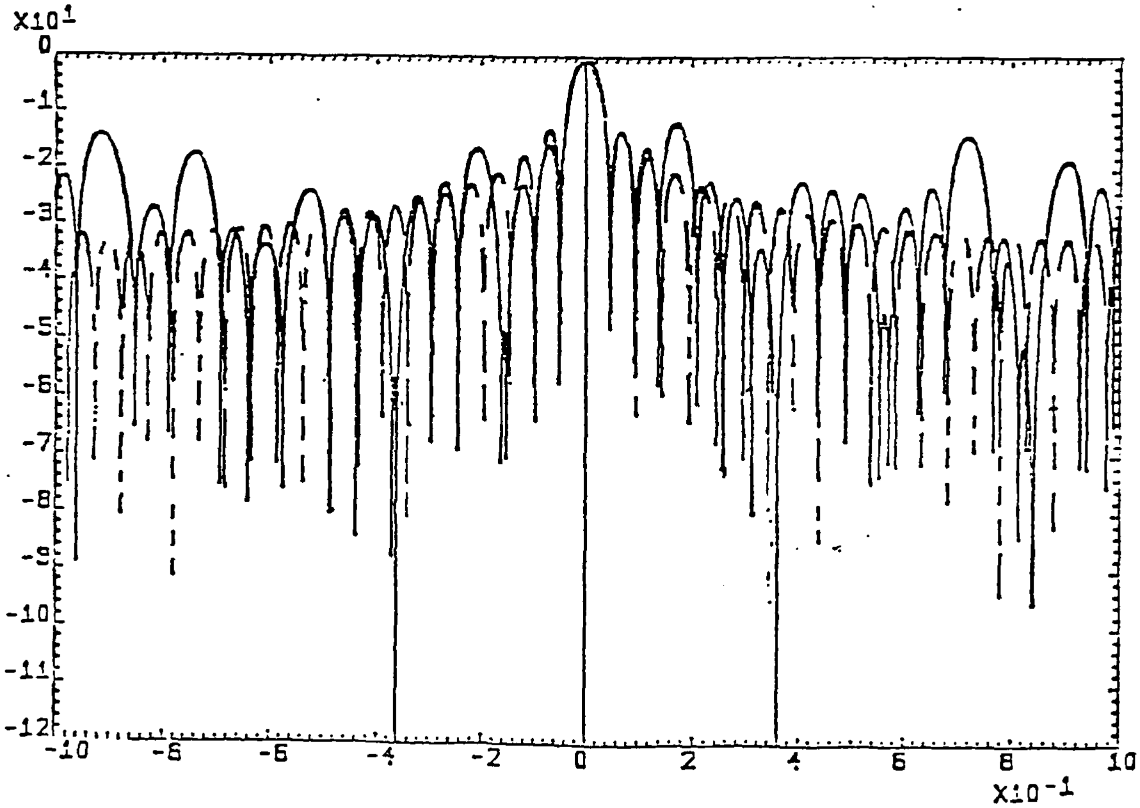
(e) Uk = ± 0.386



(f) Uk = ± 0.390



(c) Uk = ± 0.381



(d) Uk = ± 0.376

Figure 4: Increasing Null width by Higher order Constraints.

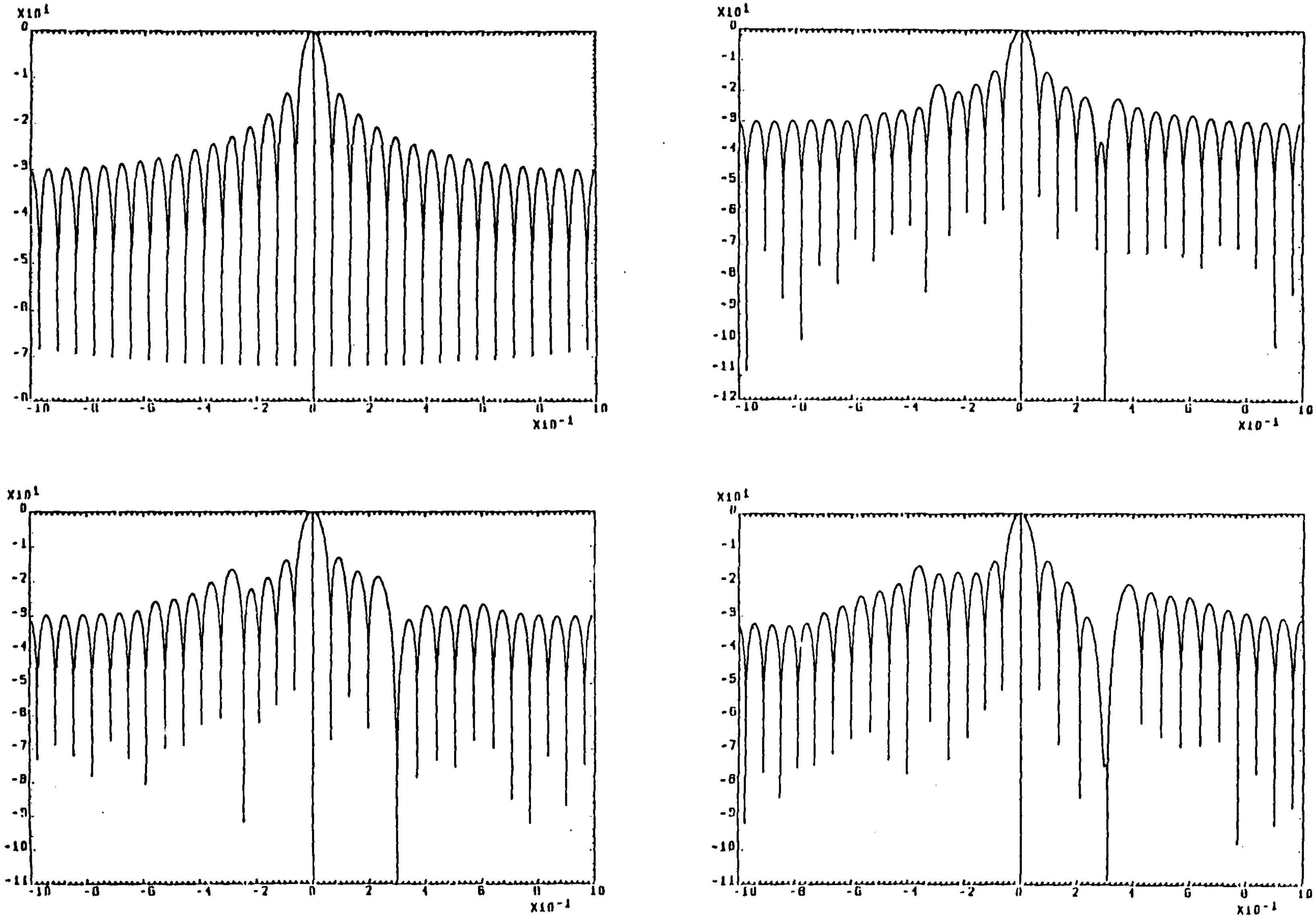


Fig. 5 Coordinate system for 2-D array

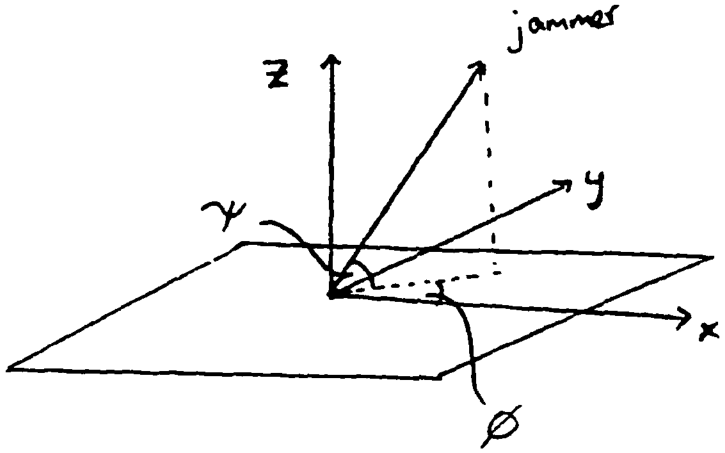


Fig. 6

Elements of 2-D array

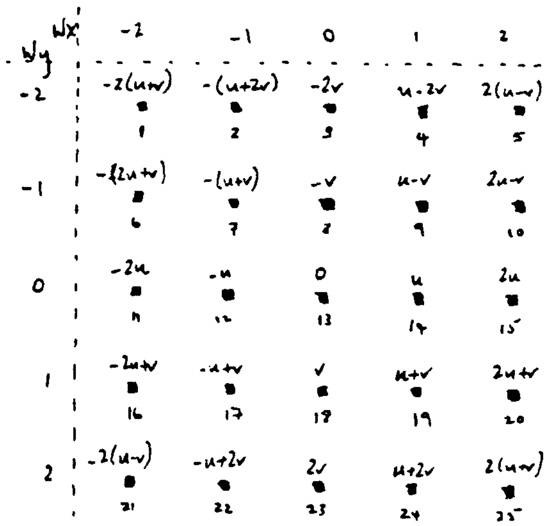


Fig. 7

Embedded octagon in 2-D array

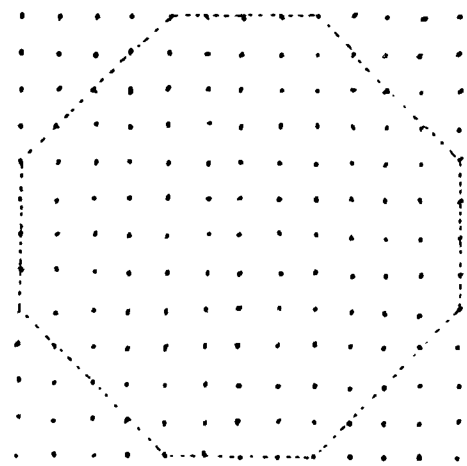


Fig. 8(a)

Field Pattern for Rectangular Array; 13x13 elements

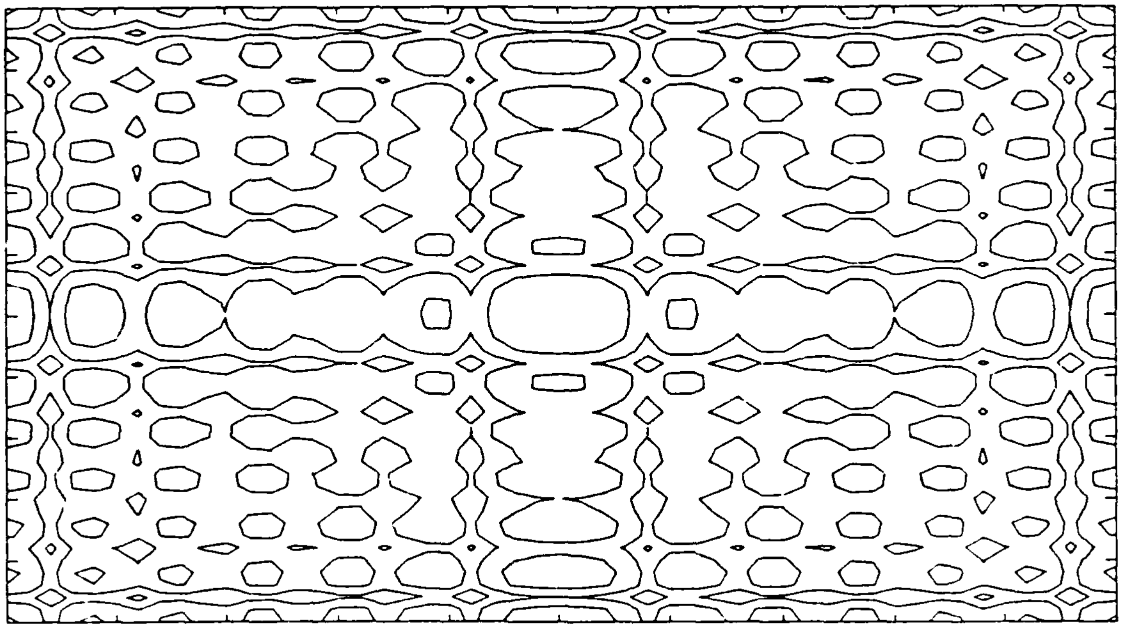


Fig. 8(b)

Field Pattern for Octagonal Array embedded in 13x13 rectangle

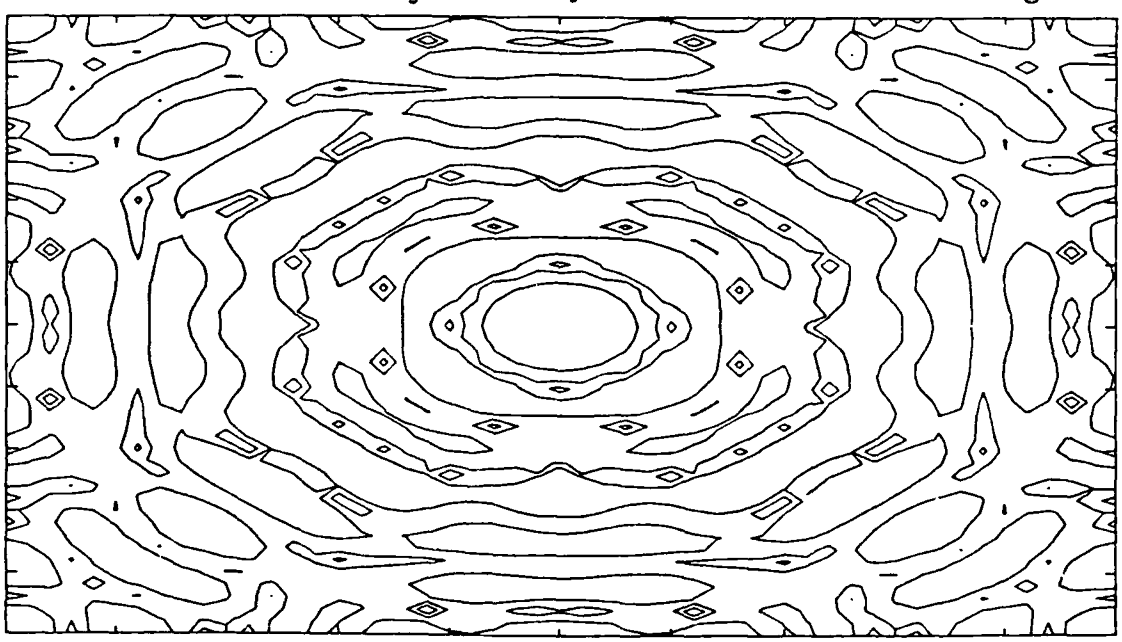


Fig. 9(a)

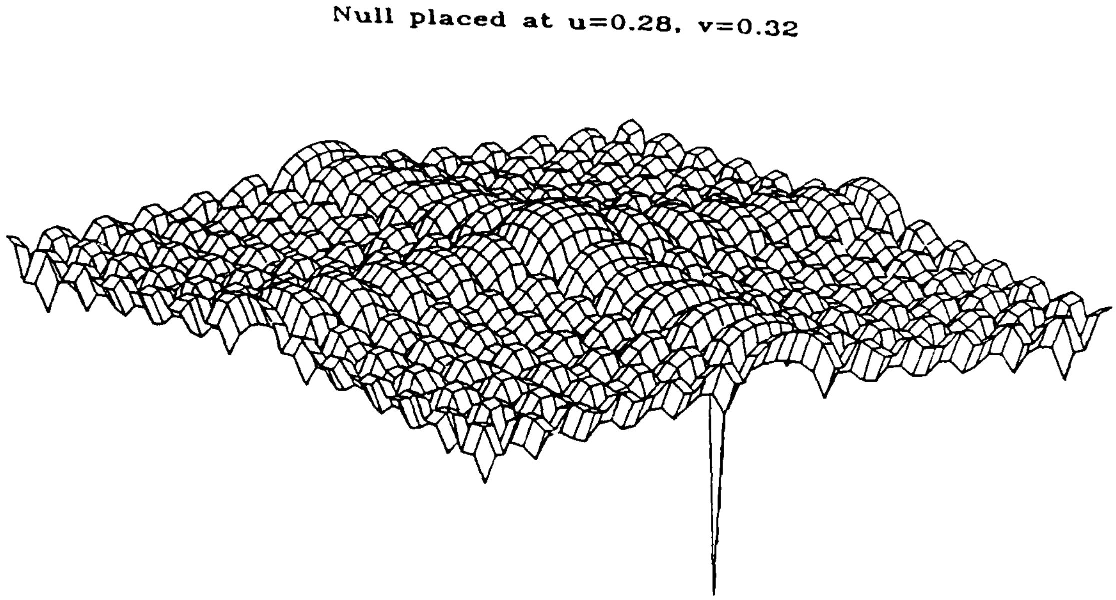

Null placed at u=0.28, v=0.32

Fig. 9(b)

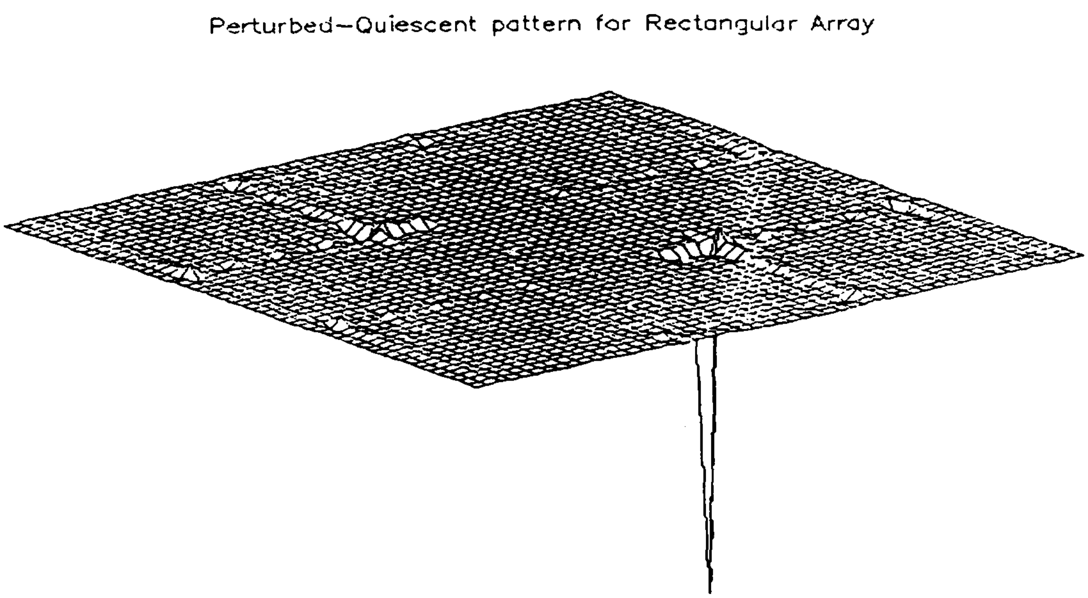

Perturbed–Quiescent pattern for Rectangular Array

Fig. 10(a)

Null placed at u=0.28, v=0.32

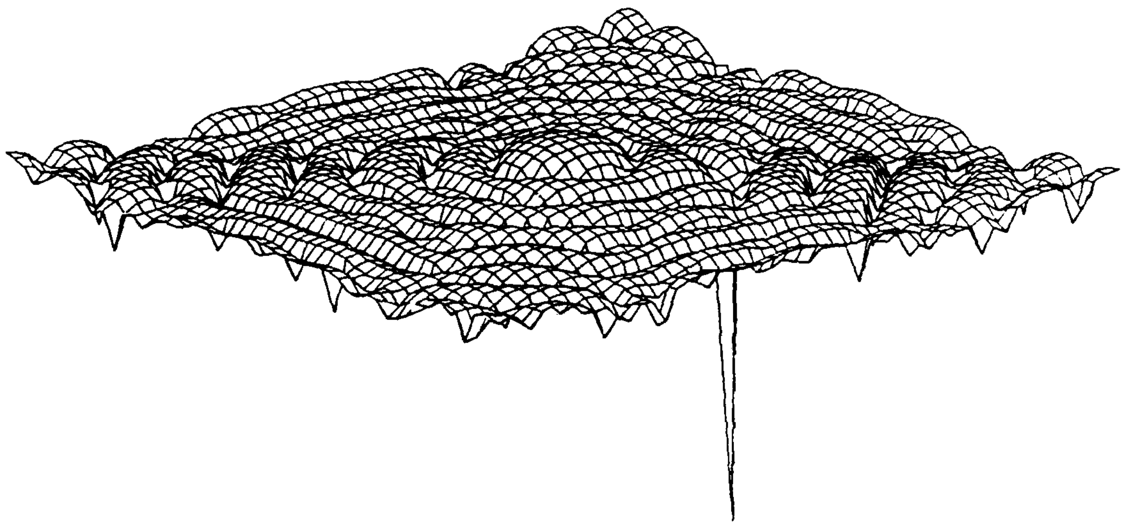


Fig. 10(b)

Perturbed-Quiescent pattern for Octagonal Array

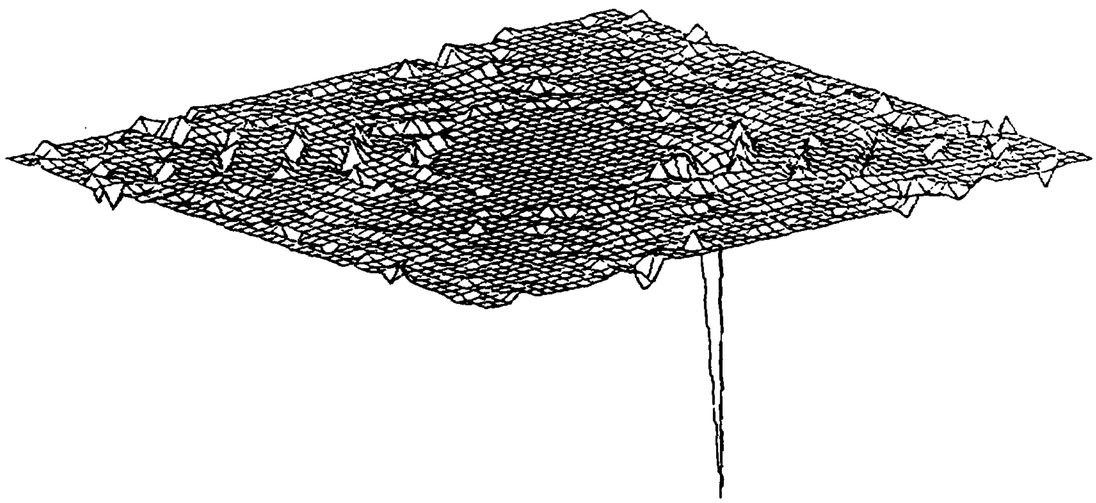


Fig. 11(a)

Nulls placed at u=0.28,v=0.32 and u=0.32,v=0.36

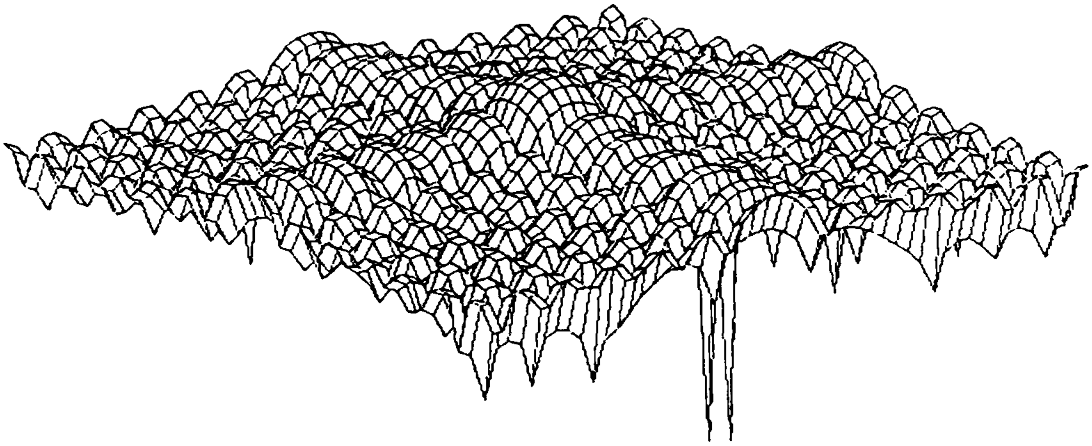


Fig. 11(b)

Perturbed-Quiescent pattern for Rectangular Array

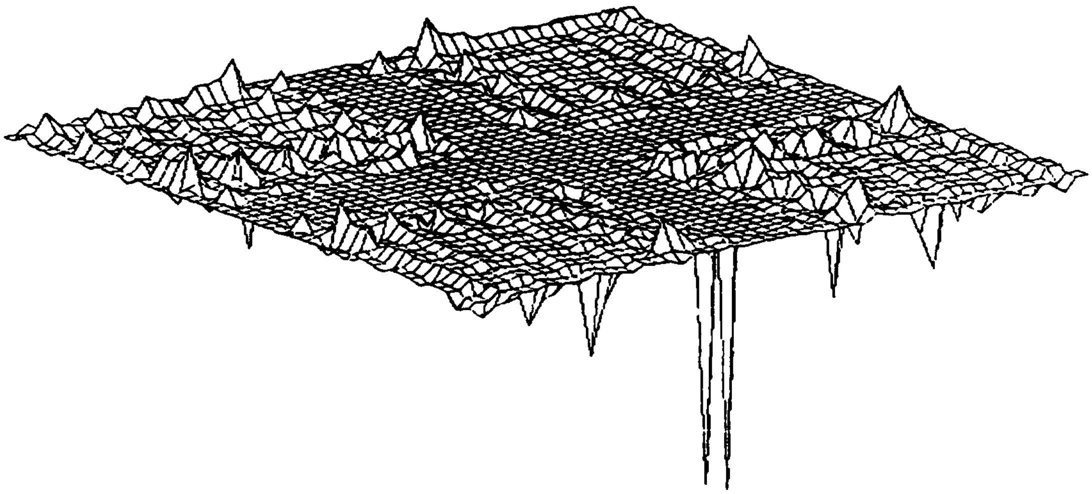


Fig. 12(a)

Nulls placed at u=0.28,v=0.32 and u=0.32,v=0.36

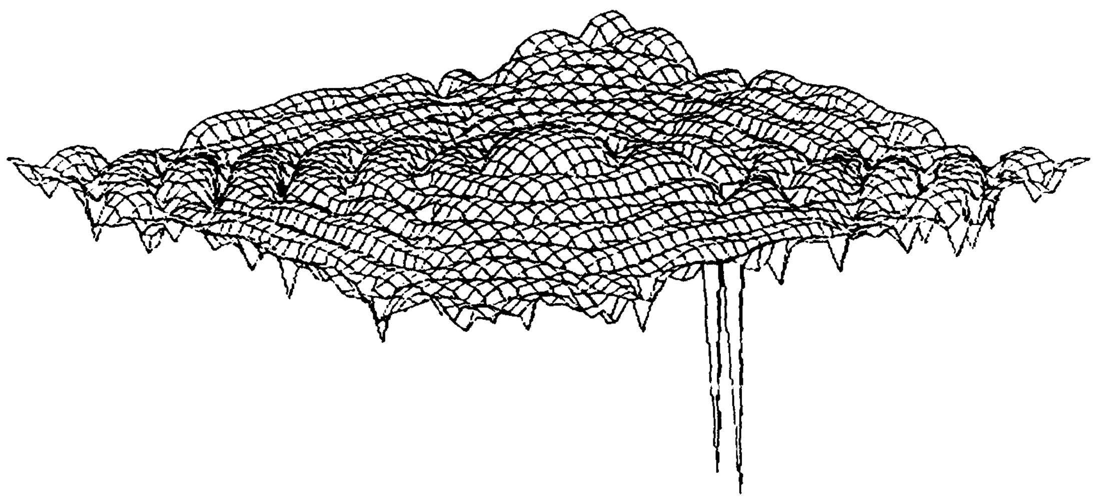


Fig. 12(b)

Perturbed–Quiescent pattern for Octagonal Array

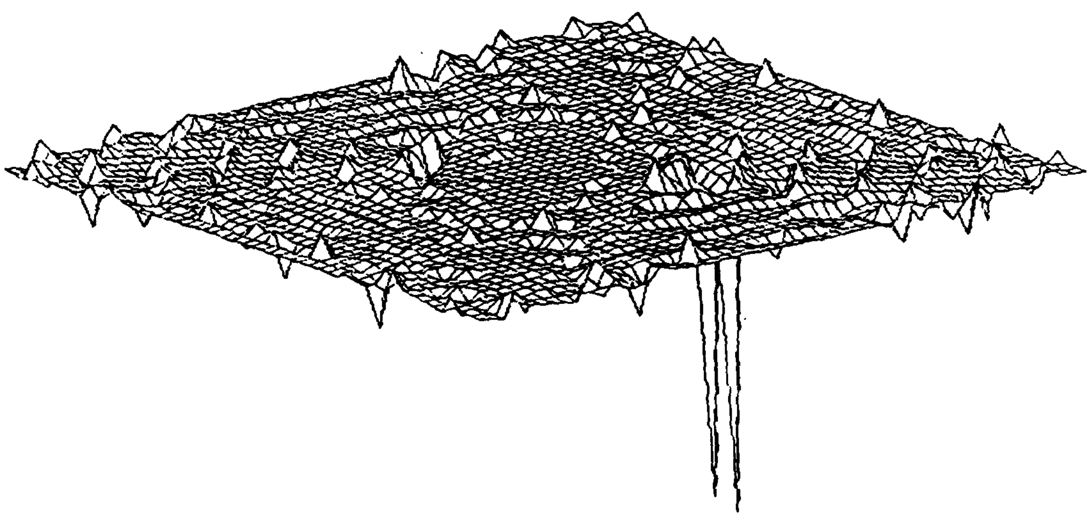


Fig. 13

Quiescent sinc pattern with 41 elements

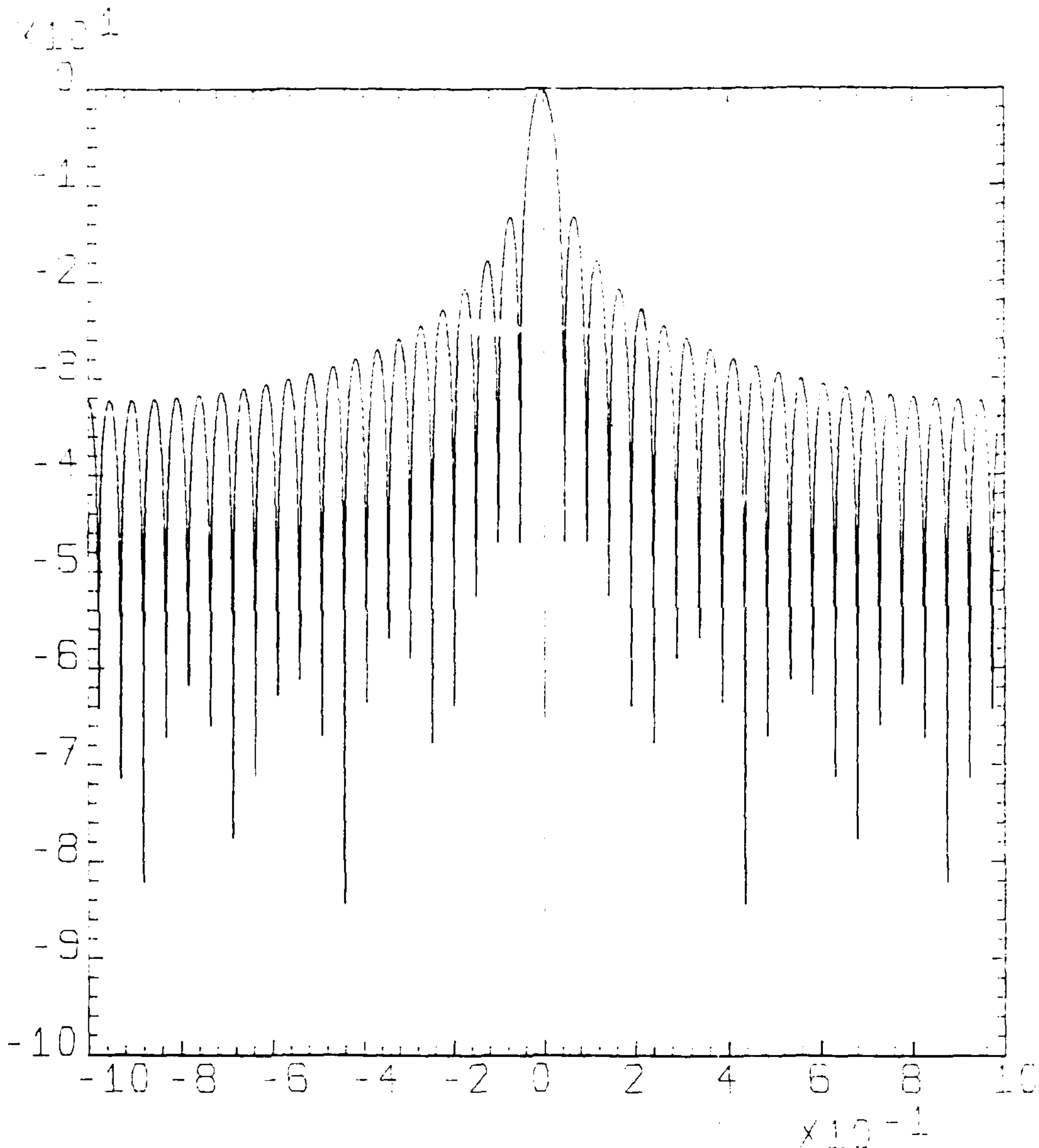


Fig. 14

Quiescent 20 db Taylor pattern with 41 elements and n=6

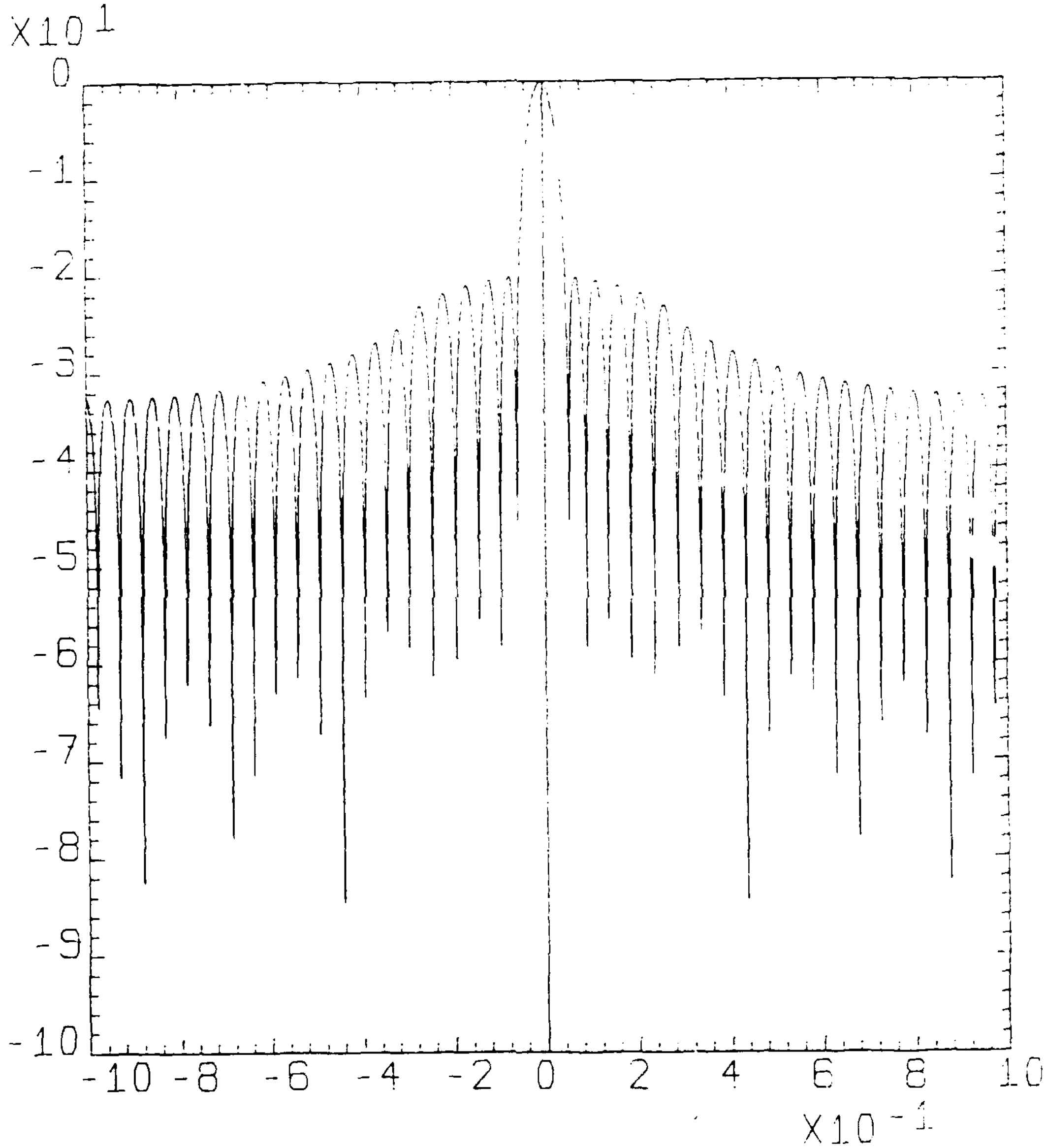


Fig. 15

Quiescent 30 db Taylor pattern with 41 elements and n=6

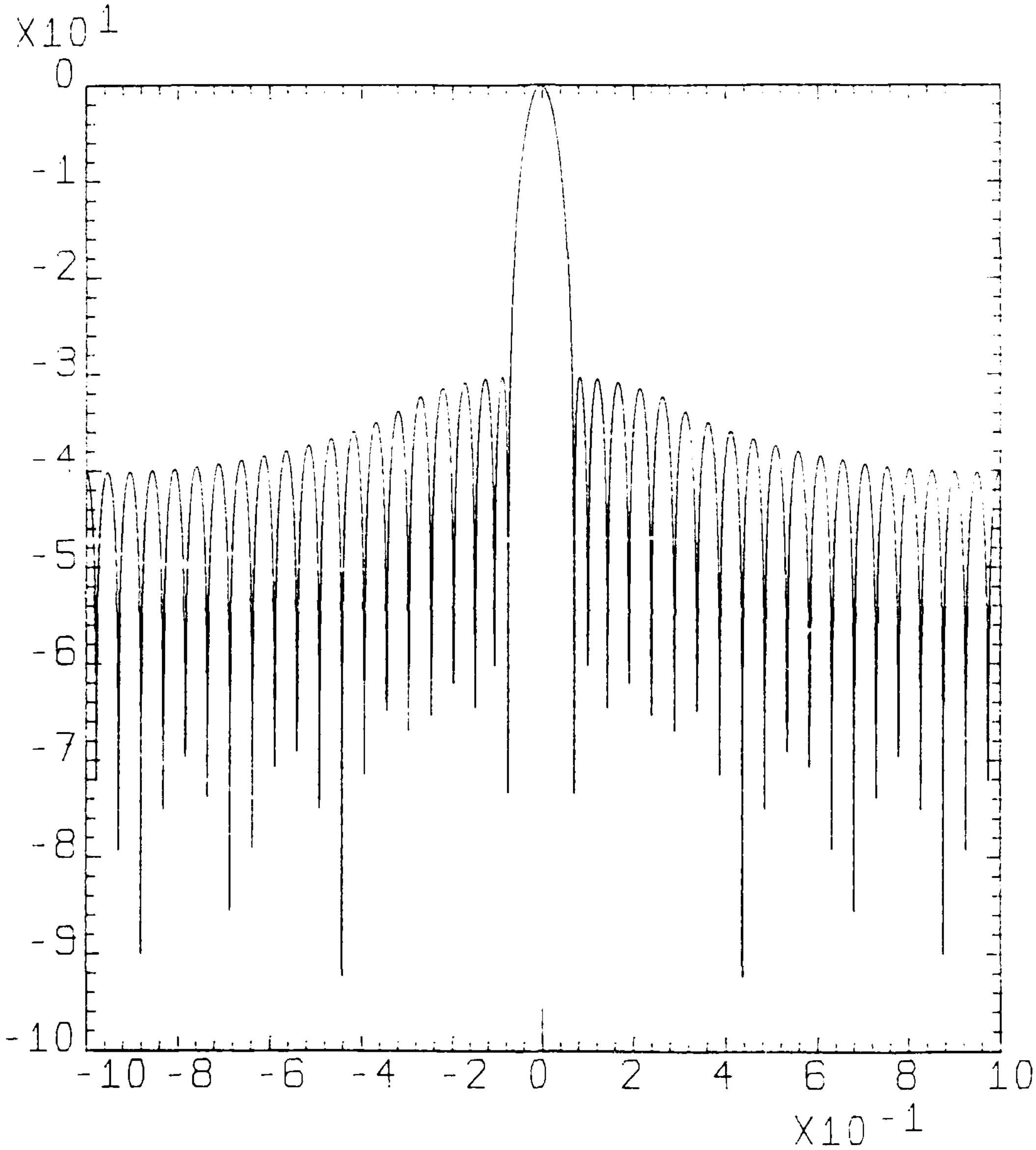


Fig. 16 Constrained amplitude-only pattern.

Nulls at u=0.7,0.71,0.72,0.73

Quiescent pattern - sinc

x_i= 1.000000000 (i=1,...,41) except

$x_1 = x_{41} = 0.27835309$

$x_2 = x_{40} = 0.65933325$

$x_3 = x_{39} = 0.96323350$

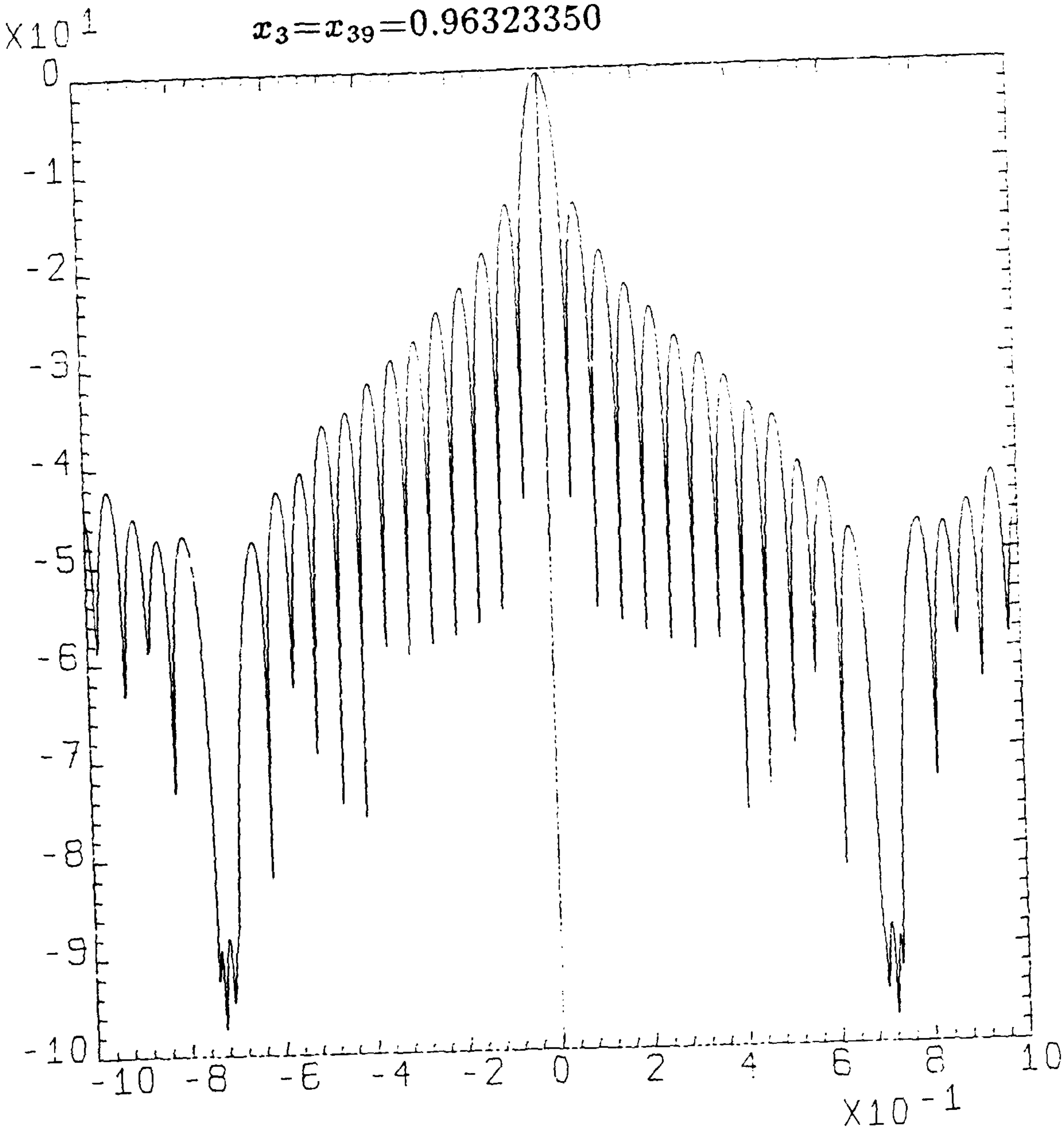


Fig. 17 Constrained amplitude-only pattern.
Nulls at u=0.07,0.08
Quiescent pattern - sinc
x_i=1.000000000 (i=1,...,41) except
$x_5=x_{37}$=-0.61903885
$x_6=x_{36}$=-0.99823899

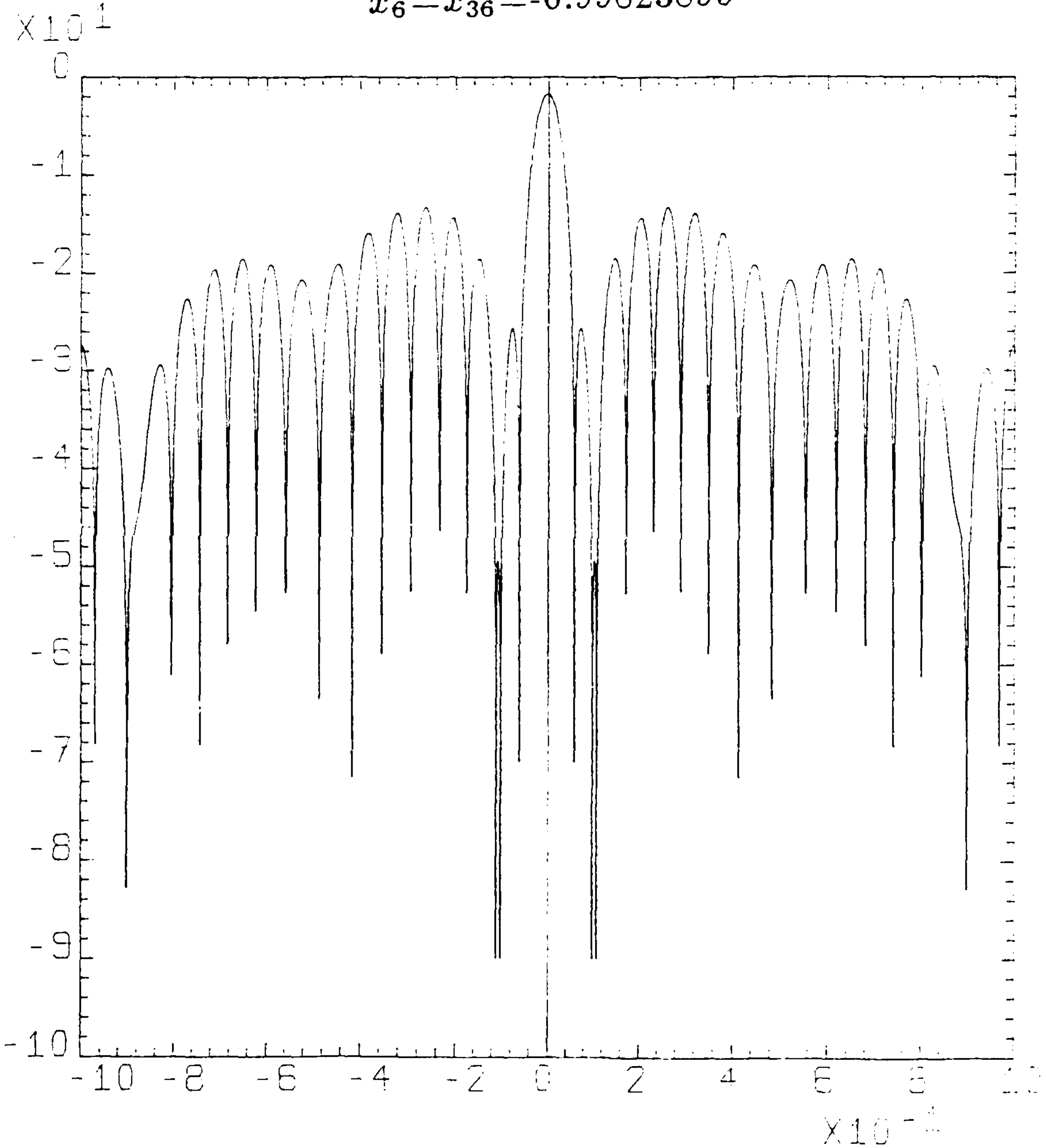


Fig. 18

Constrained phase-only pattern. Nulls at u=0.07,0.08
Phase range [-Π,Π]

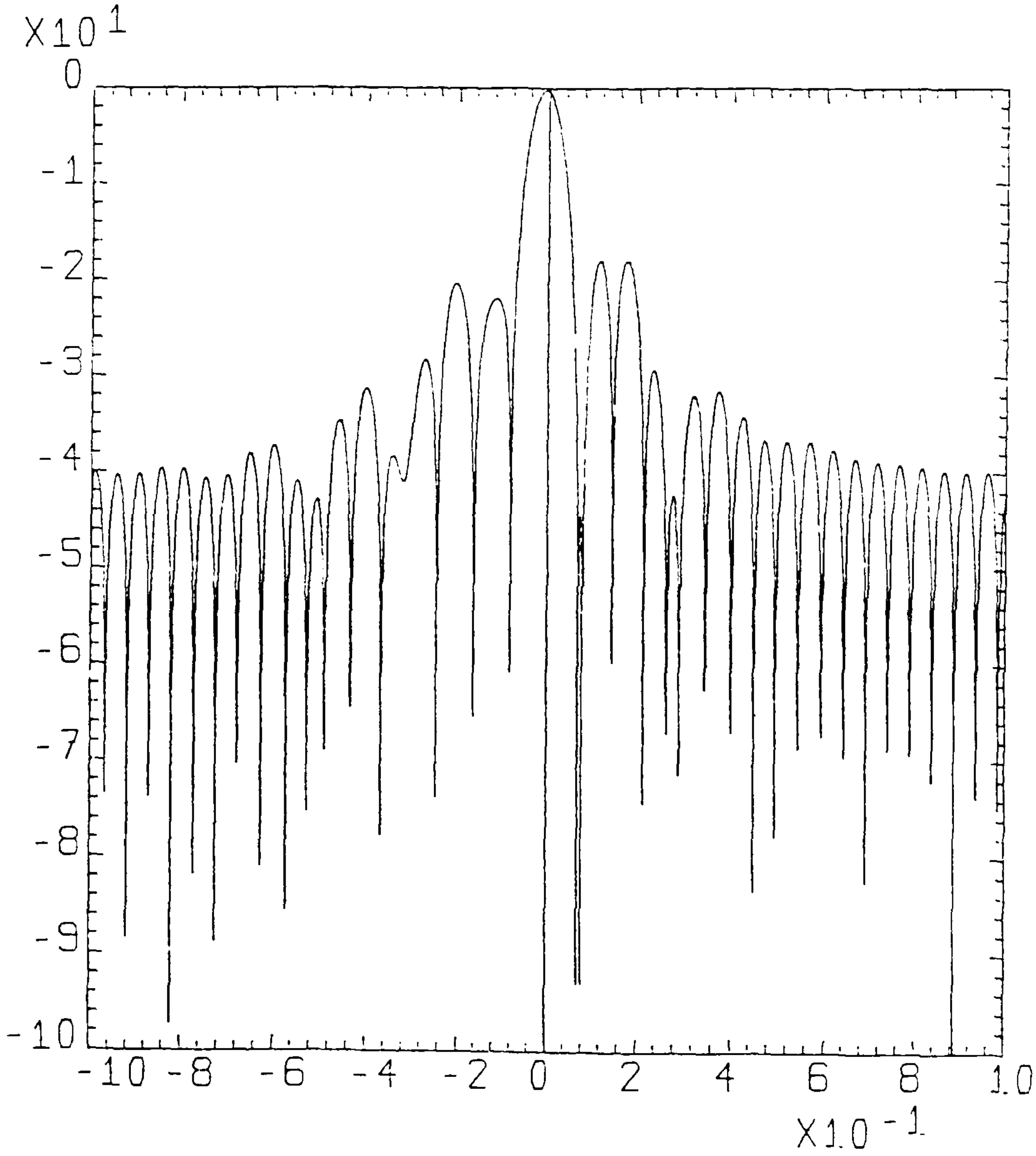


Fig. 19

Constrained phase-only pattern. Nulls at u=0.4,0.525, 0.65,0.775,0.9. Phase range [-Π,Π]

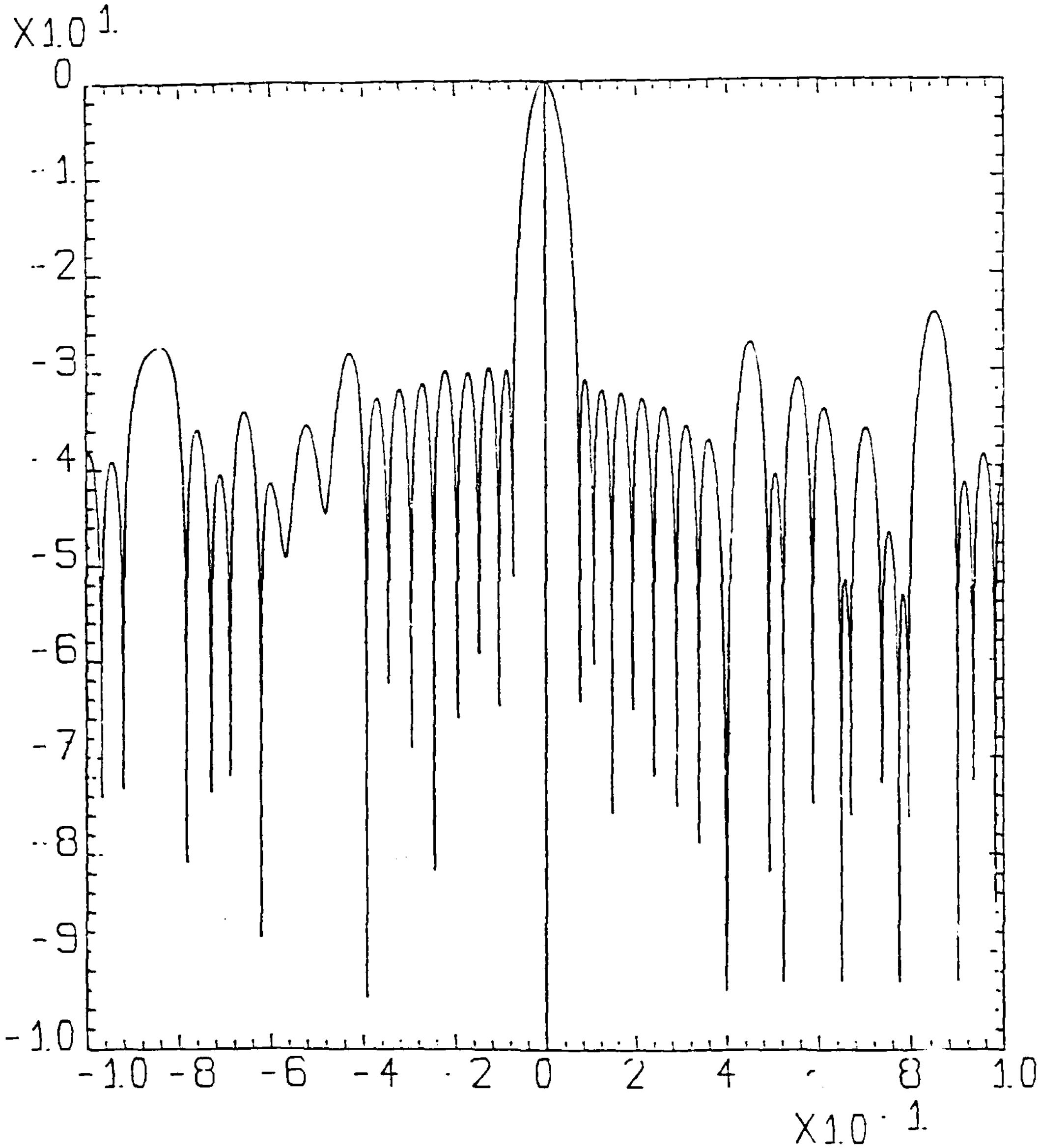


Fig. 20

Constrained phase-only pattern. Nulls at u=0.22,0.24, 0.26,0.28,0.3,0.32,0.34,0.36. Phase range [-Π,Π]

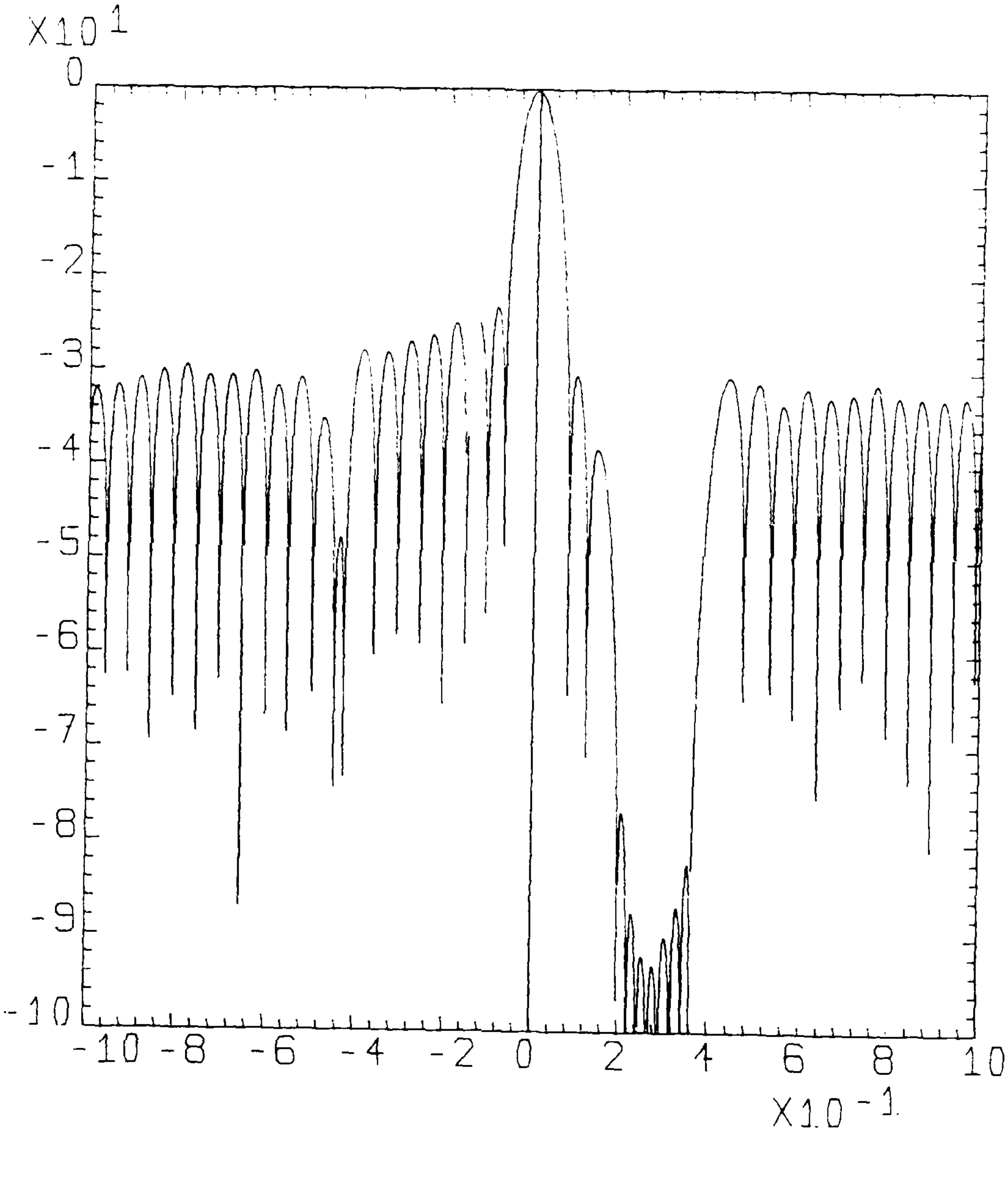


Fig. 21

Constrained phase-only pattern. Nulls at u=0.22,0.24, 0.26,0.28. Phase range [-Π,Π]

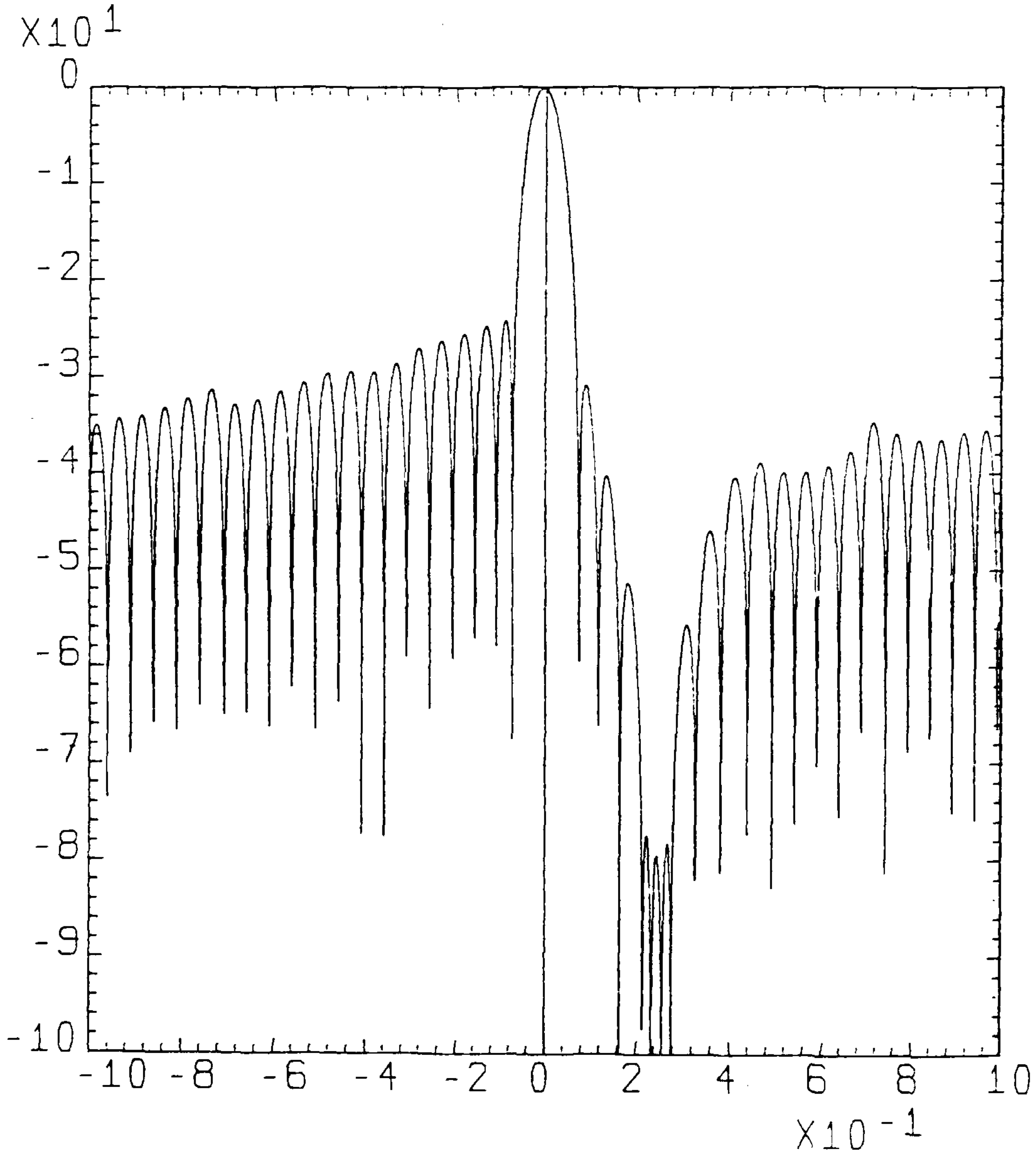


Fig. 22

Constrained phase-only pattern. Nulls at u=0.22,0.24, 0.26,0.28. Phase range [0,2Π]

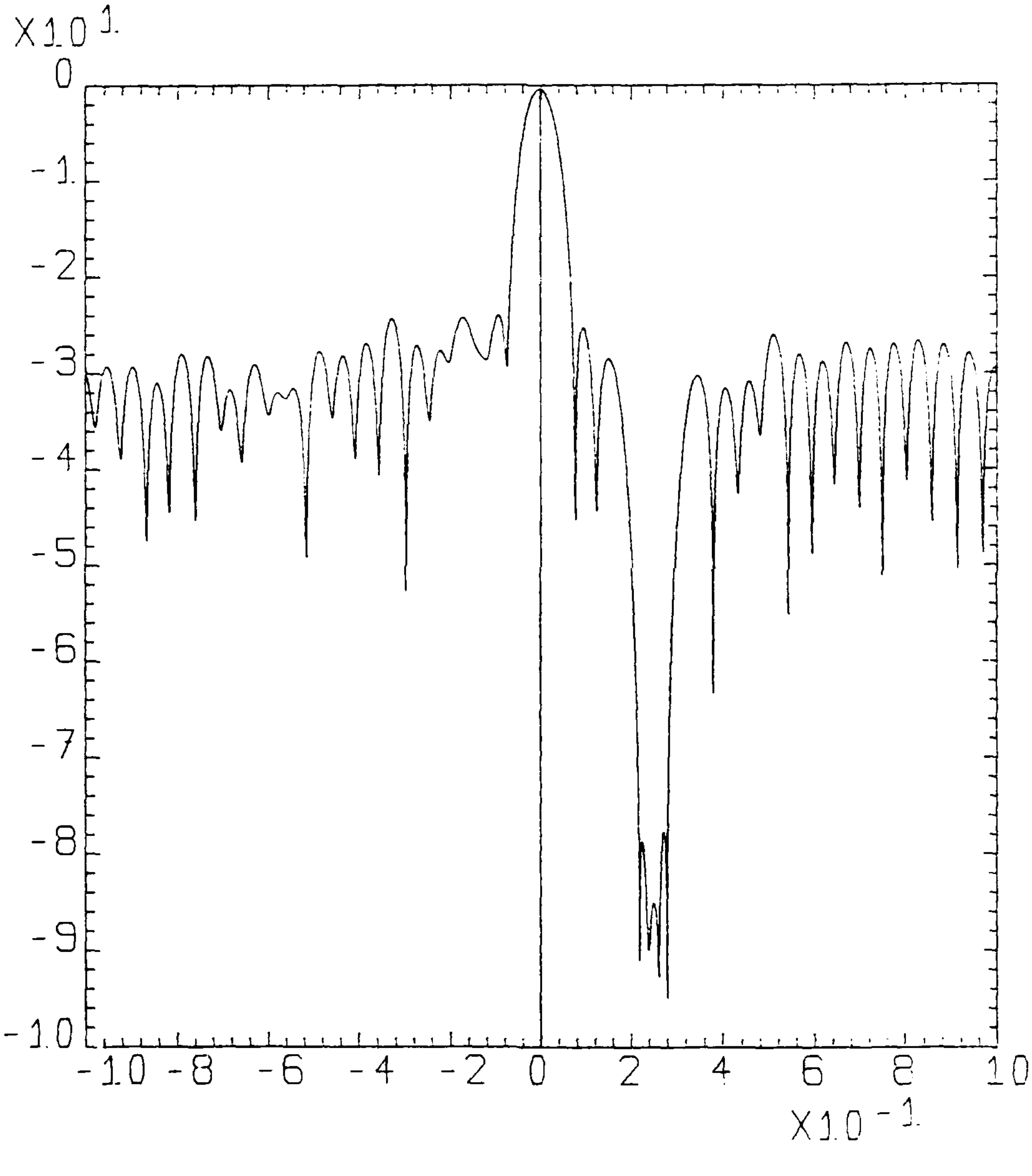


Fig. 23(a)

Constrained phase-only pattern. Constraints over u=[0.7,0.72]

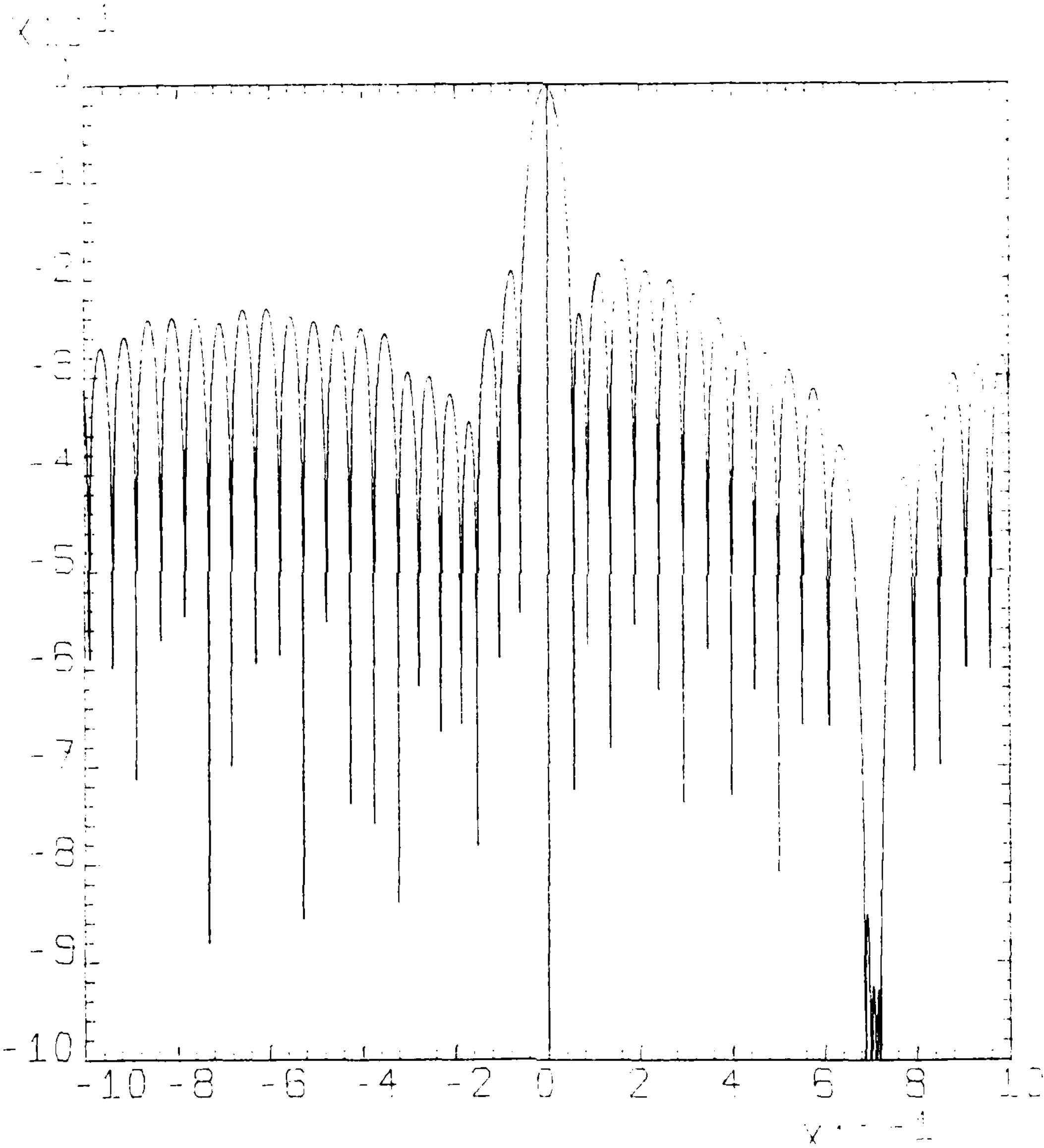


Fig. 23(b) Array element 4 failed

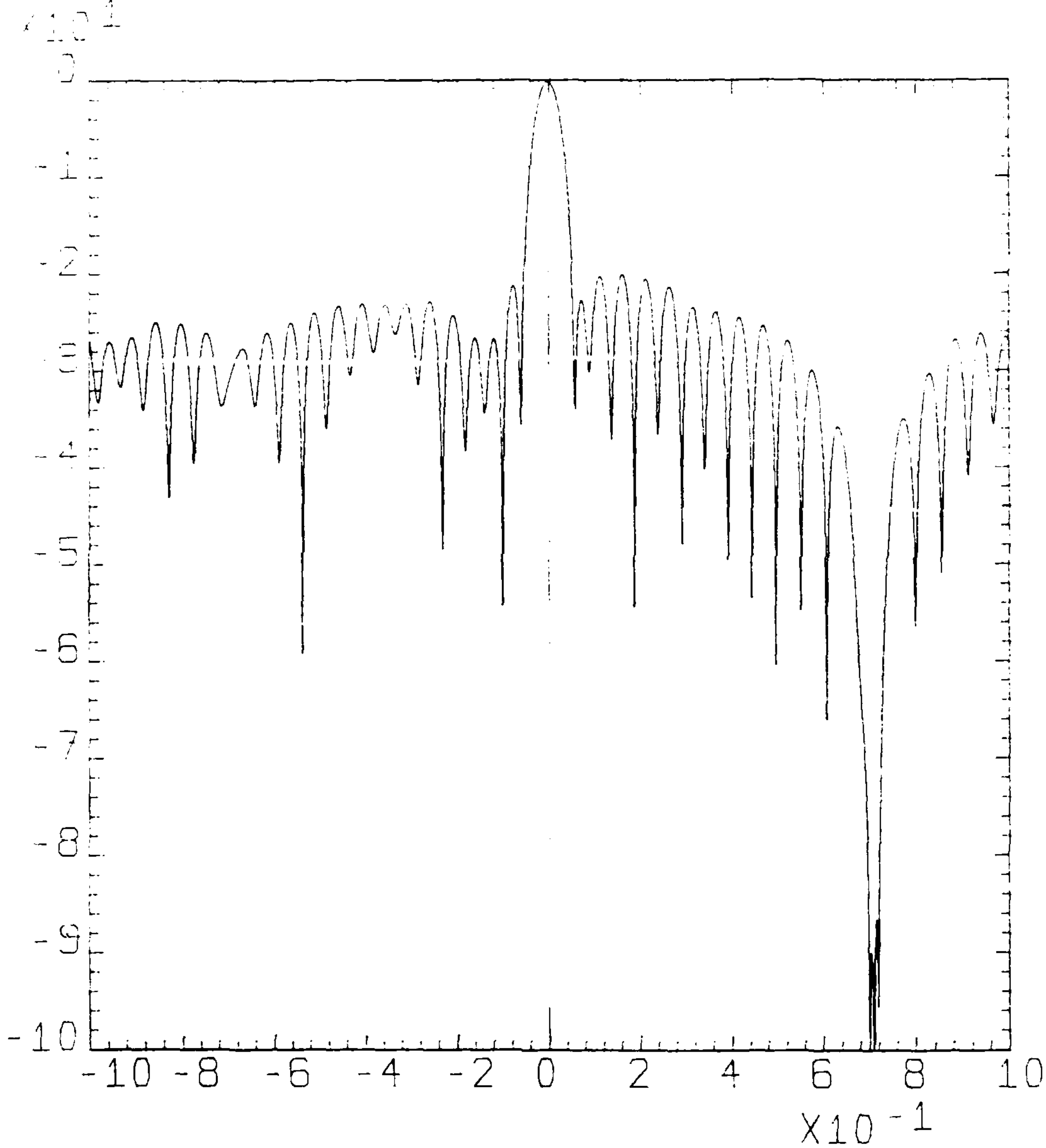


Fig. 23(c) Array elements 4,11 failed

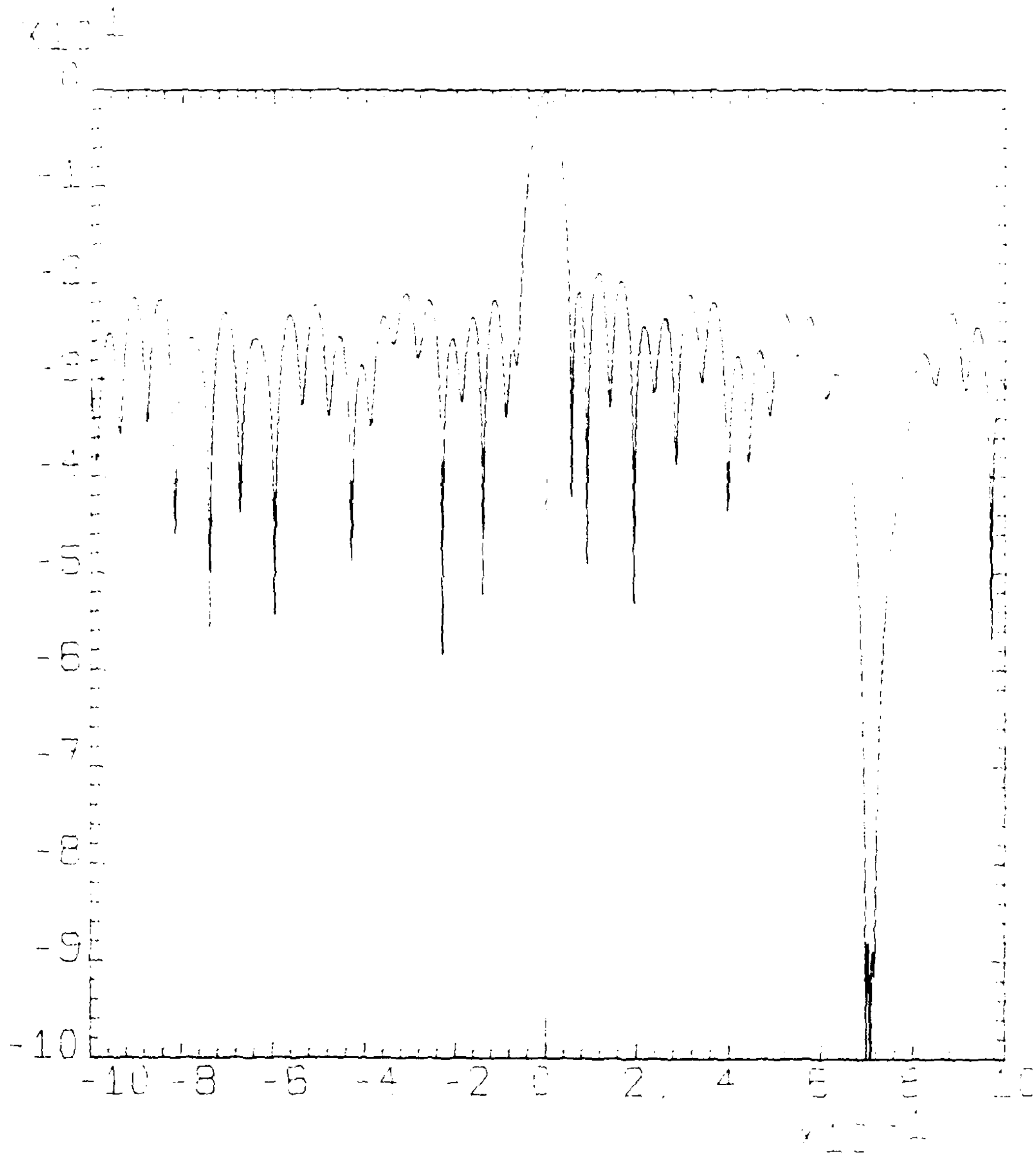
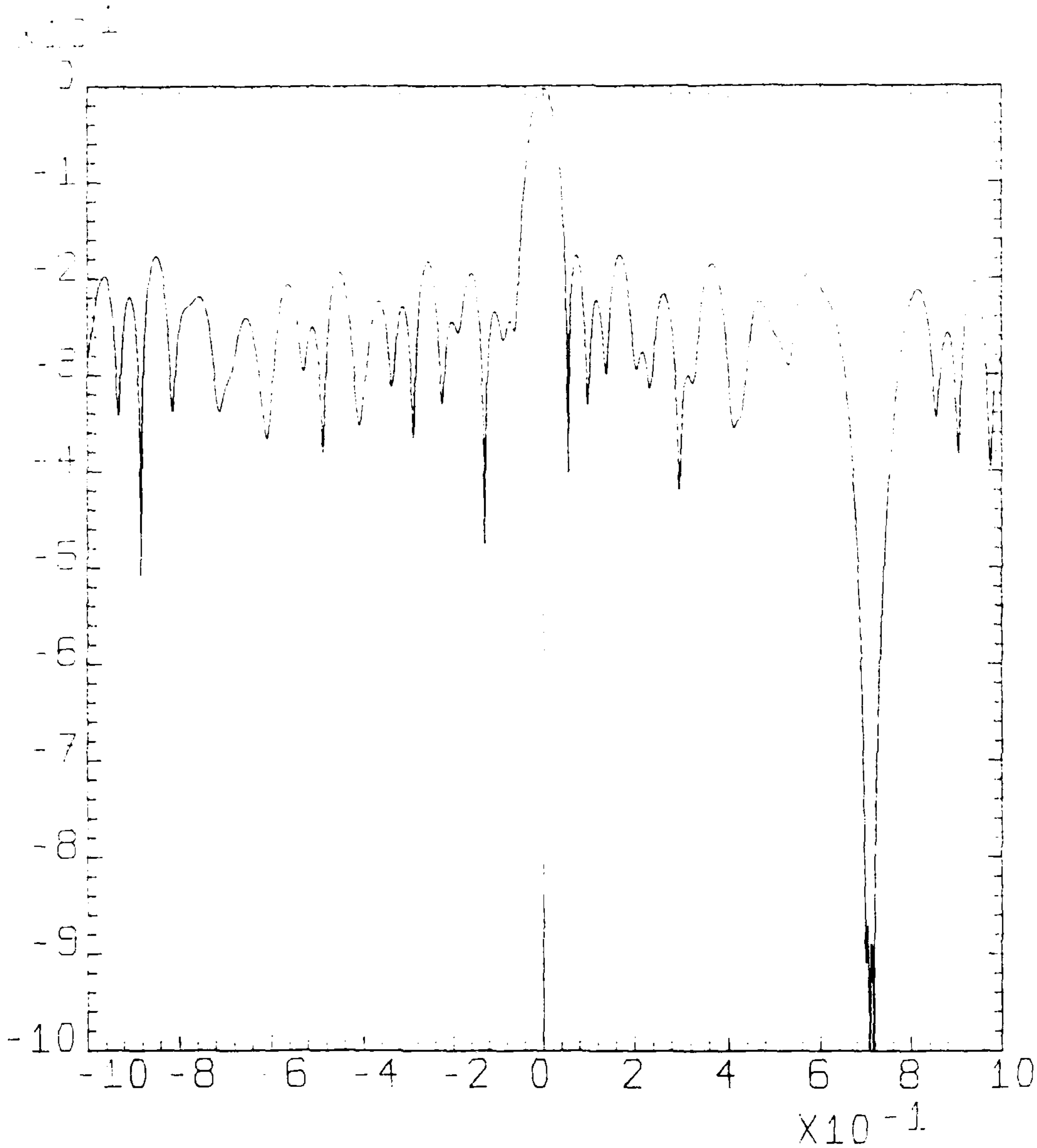


Fig. 23(d) Array elements 4,11,21 failed



List of Tables

Table 1a: Results for Null at Uk = 0.43633

Null direction (rad)	Optimal Beam Coefficients	Average Phase Pertubation (rad)	Null Depth (DBA)	Loss at Main Beam
0.43633	0.13468	4.32306E-02	-334.54672	3.902157

Table 1b: NAG output for above example

```
WORKSPACE PROVIDED IS      IW(      4),  W(    110).
TO SOLVE PROBLEM WE NEED   IW(      4),  W(    110).

EXIT NP PHASE.             IFAIL =     0
      NO. OF MAJOR ITERATIONS =     3
  NO. OF FUNCTION EVALUATIONS =     5
NO. OF CONSTRAINT EVALUATIONS =     5

VARBL STATE     VALUE       LOWER BOUND    UPPER BOUND     LAGR MULT    RESIDUAL

V  1    FR  0.1346774      -1000.000       1000.000        0.0000E+00    999.9

NLCON STATE     VALUE       LOWER BOUND    UPPER BOUND     LAGR MULT    RESIDUAL

N  1    EQ -0.7978722E-10  0.0000000E+00  0.0000000E+00   0.1347       -0.7979E-10

EXIT E04VDF - OPTIMAL SOLUTION FOUND.

OBJECTIVE FUNCTION                 =    9.522983D-02
NORM OF THE CONSTRAINT VIOLATIONS  =    7.978722D-11

NO. OF ITERATIONS =     3
```

Table 2a: Results obtained for Multiple Nulls

Null direction (rad)	Optimal Beam Coefficients	Average Phase Pertubation (rad).	Null Depth (DBA)	Loss at Main Beam
0.59556	-0.12590	4.01644E-02	-300.86929	3.3491
0.59556 0.65575	-0.12430 -0.11685	4.96558E-02	-320.59109 -323.25893	6.2658
0.59556 0.65575 0.71887	-0.12161 -0.11331 -0.10434	5.28214E-02	-282.87559 -280.68316 -275.63945	8.7066

Table 2b: Effect of Close Null Locations

Null direction (rad)	Optimal Beam Coefficients	Average Phase Pertubation (rad)	Null Depth (DBA)	Loss at Main Beam
0.59556 0.64046	-0.16944 0.14218	5.22363E-02	-297.34205 -307.38120	7.08540
0.59556 0.62533	-0.22427 0.14954	4.94901E-02	-288.81105 -309.78194	5.96120
0.59556 0.61037	-0.26831 0.15516	4.42074E-02	-227.86140 -225.92093	4.25921
0.59556 0.60295	-0.29746 -0.17432	4.14764E-02	-252.33825 -258.54909	3.66009

Table 3: Coefficients and Average pertubations for Higher order Constraints.

Order of Constraint	Coefficients	Average Pertubation (rad)
0	- 0.2435	7.8994e-2
1	- 0.2410 - 3.1956e-2	0.1128
2	- 0.2825 - 3.1150e-2 - 6.0153e-3	0.1532

Table 4 Results for Rectangular Array with one Null.

Antenna Arrays : Phase Only Nulling

There are 169 elements and 1 contraints.

Constraint 1 is at position .28000 .32000

Initial value for constraint 1 is .10000

Required accuracy is .10000E-07

Optimal Beam Coefficients

.15315E-01

Coefficients for Perturbed pattern

1	.72655E-02
2	.59292E-02
3	-.96710E-03
4	-.69514E-02
5	-.64391E-02
6	-.48876E-17
7	.64391E-02
8	.69514E-02
9	.96710E-03
10	-.59292E-02
11	-.72655E-02
12	-.18904E-02
13	.52129E-02
14	.64921E-02
15	.57311E-17
16	-.64921E-02
17	-.69062E-02
18	-.95252E-03
19	.58716E-02
20	.73000E-02
21	.19186E-02
22	-.52714E-02
23	-.75111E-02
24	-.27990E-02
25	.44733E-02
26	.76461E-02
27	.96710E-03
28	-.59292E-02
29	-.72655E-02
30	-.18904E-02
31	.52129E-02
32	.75327E-02
33	.28392E-02
34	-.45291E-02
35	-.76387E-02
36	-.36645E-02
37	.36645E-02
38	.76387E-02
39	.45291E-02
40	-.52714E-02
41	-.75111E-02
42	-.27990E-02
43	.44733E-02
44	.76461E-02
45	.37140E-02
46	-.37140E-02
47	-.76461E-02
48	-.44733E-02
49	.27990E-02
50	.75111E-02
51	.52714E-02
52	-.19186E-02
53	-.76387E-02
54	-.36645E-02
55	.36645E-02
56	.76387E-02
57	.45291E-02
58	-.28392E-02
59	-.75327E-02
60	-.52129E-02
61	.18904E-02
62	.72655E-02
63	.59292E-02
64	-.96710E-03
65	-.69514E-02
66	-.44733E-02
67	.27990E-02
68	.75111E-02
69	.52714E-02
70	-.19186E-02
71	-.73000E-02
72	-.58716E-02
73	.95252E-03
74	.69062E-02
75	.64921E-02
76	-.76805E-18
77	-.64921E-02
78	-.69062E-02
79	.18904E-02
80	.72655E-02
81	.59292E-02
82	-.96710E-03
83	-.69514E-02
84	-.64391E-02
85	.00000

Average pertubation = .48752E-02

Quisecent pattern at uk = .28000 .32000
Real = -.64964 Imag = .51033E-15

Perturbed pattern at uk = .28000 .32000
Real = .12688E-09 Imag = -.11861E-15

Null depth = -240.14 DBA

Table 5 Results for Octagonal Array with one Null.

Antenna Arrays : Phase Only Nulling

There are 129 elements and 1 contraints.

Constraint 1 is at position .28000 .32000

Initial value for constraint 1 is .10000

Required accuracy is .10000E-07

Optimal Beam Coefficients

-.27348

Coefficients for Perturbed pattern

5	.12394
6	.10187E-15
7	-.12394
8	-.11639
9	-.15089E-01
17	.13062
18	.19825E-01
19	-.11491
20	-.12413
21	-.30019E-01
22	.84914E-01
23	.13698
29	.13496
30	.39176E-01
31	-.10359
32	-.13022
33	-.44629E-01
34	.72241E-01
35	.13679
36	.74702E-01
37	-.74702E-01
41	.13698
42	.57604E-01
43	-.90124E-01
44	-.13449
45	-.58757E-01
46	.58757E-01
47	.13449
48	.90124E-01
49	-.57604E-01
50	-.13698
51	-.84914E-01
53	.13679
54	.74702E-01
55	-.74702E-01
56	-.13679
57	-.72241E-01
58	.44629E-01
59	.13022
60	.10359
61	-.39176E-01
62	-.13496
63	-.96610E-01
64	.15089E-01
65	.11639
66	.90124E-01
67	-.57604E-01
68	-.13698
69	-.84914E-01
70	.30019E-01
71	.12413
72	.11491
73	-.19825E-01
74	-.13062
75	-.10716
76	.11972E-16
77	.10716
78	.13062
79	-.39176E-01
80	-.13496
81	-.96610E-01
82	.15089E-01
83	.11639
84	.12394
85	.00000

Average pertubation = .67559E-01

Quisecent pattern at uk = .28000 .32000
Real = 9.0426 Imag = -.25479E-16

Perturbed pattern at uk = .28000 .32000
Real = .98096E-15 Imag = .14154E-15

Null depth = -342.29 DBA

Table 6 Results for Rectangular Array with 2 Nulls.

Antenna Arrays : Phase Only Nulling

There are 169 elements and 2 contraints.

Constraint 1 is at position .28000 .32000
Constraint 2 is at position .32000 .36000

Initial value for constraint 1 is .10000
Initial value for constraint 2 is .10000

Required accuracy is .10000E-07
Optimal Beam Coefficients
.86483E-01
-.89406E-01

Coefficients for Perturbed pattern

1	.53802E-01
2	-.11934E-02
3	-.46955E-01
4	-.38105E-01
5	.37794E-02
6	.33931E-01
7	.25908E-01
8	-.49172E-02
9	-.20995E-01
10	-.11646E-01
11	.34537E-02
12	.57008E-02
13	-.10019E-02
14	.62571E-02
15	-.43776E-01
16	-.41216E-01
17	-.13255E-02
18	.31713E-01
19	.28181E-01
20	-.14293E-02
21	-.20031E-01
22	-.13013E-01
23	.21166E-02
24	.56119E-02
25	-.86034E-03
26	-.75122E-03
27	-.39847E-01
28	-.43719E-01
29	-.64120E-02
30	.29024E-01
31	.29987E-01
32	.20832E-02
33	-.18744E-01
34	-.14180E-01
35	.74719E-03
36	.54348E-02
37	-.70521E-03
38	-.14511E-02
39	.89014E-02
40	-.45568E-01
41	-.11409E-01
42	.25907E-01
43	.31302E-01
44	.55600E-02
45	-.17150E-01
46	-.15130E-01
47	-.63361E-03
48	.51725E-02
49	-.53892E-03
50	-.21284E-02
51	.80033E-02
52	.15610E-01
53	-.16247E-01
54	.22414E-01
55	.32112E-01
56	.89418E-02
57	-.15275E-01
58	-.15846E-01
59	-.20047E-02
60	.48291E-02
61	-.36409E-03
62	-.27727E-02
63	.69819E-02
64	.16175E-01
65	.56081E-02
66	.18598E-01
67	.32407E-01
68	.12171E-01
69	-.13147E-01
70	-.16315E-01
71	-.33453E-02
72	.44103E-02
73	-.18350E-03
74	-.33739E-02
75	.58532E-02
76	.16483E-01
77	.82735E-02
78	-.17961E-01
79	.32191E-01
80	.15194E-01
81	-.10800E-01
82	-.16529E-01
83	-.46347E-02
84	.39226E-02
85	.00000

Average pertubation = .14419E-01

Quisecent pattern at uk = .28000 .32000
Real = -.64964 Imag = .51033E-15

Perturbed pattern at uk = .28000 .32000
Real = -.19110E-07 Imag = .15856E-15

Null depth = -196.59 DBA

Quisecent pattern at uk = .32000 .36000
Real = .84424 Imag = .34094E-15

Perturbed pattern at uk = .32000 .36000
Real = -.20194E-07 Imag = -.12334E-14

Null depth = -196.11 DBA

Table 7 Results for Octagonal Array with two Nulls.

Antenna Arrays : Phase Only Nulling

There are 129 elements and 2 contraints.

Constraint 1 is at position .28000 .32000
Constraint 2 is at position .32000 .36000

Initial value for constraint 1 is .10000
Initial value for constraint 2 is .10000

Required accuracy is .10000E-07
Optimal Beam Coefficients
-.59158
.37090

Coefficients for Perturbed pattern

5	.10705
6	-.17203
7	-.19683
8	-.73508E-01
9	.62827E-01
17	.14239
18	-.14722
19	-.20278
20	-.91101E-01
21	.46339E-01
22	.12795
23	.11257
29	.17337
30	-.11836
31	-.20582
32	-.10779
33	.29386E-01
34	.11997
35	.11885
36	.37580E-01
37	-.58779E-01
41	.19944
42	-.86054E-01
43	-.20569
44	-.12337
45	.12141E-01
46	.11046
47	.12300
48	.52316E-01
49	-.45232E-01
50	-.10985
51	-.10702
53	.22033
54	-.51102E-01
55	-.20213
56	-.13764
57	-.52235E-02
58	.99605E-01
59	.12505
60	.65926E-01
61	-.30714E-01
62	-.10472
63	-.11400
64	-.46184E-01
65	.72443E-01
66	-.14530E-01
67	-.19490
68	-.15037
69	-.22537E-01
70	.87578E-01
71	.12508
72	.78154E-01
73	-.15528E-01
74	-.97679E-01
75	-.11943
76	-.60697E-01
77	.56321E-01
78	.17034
79	-.18386
80	-.16135
81	-.39627E-01
82	.74551E-01
83	.12316
84	.88790E-01
85	.00000

Average pertubation = .80078E-01

Quisecent pattern at uk = .28000 .3200(
Real = 9.0426 Imag = -.25479E-16

Perturbed pattern at uk = .28000 .3200(
Real = -.46413E-11 Imag = -.46632E-15

Null depth = -268.88 DBA

Quisecent pattern at uk = .32000 .3600(
Real = 4.7164 Imag = -.11891E-14

Perturbed pattern at uk = .32000 .3600(
Real = -.58462E-12 Imag = .46453E-15

Null depth = -286.87 DBA

Table 8 Effect of 2 near boresight constraints on 30 db Taylor pattern. Phase range [-Π,Π] (Fig. 18)

```
BEAM REFERENCE AT CENTRE OF ARRAY
INITIAL PATTERN TAYLOR WEIGHTED
NUMBER OF EQUAL SIDELOBES                  =  6
DEPTH OF SIDELOBES                         =  30.0
NUMBER OF ELEMENTS                         =   41
NUMBER OF FITTING POINTS                   =   41
DEPTH OF NULL                              =  -90.
NULL INTERVAL                              = [0.070,0.080]
SCALING FACTOR                             = 1.000
EPSI(NULL DEPTH x SCALING FACTOR)= .5927E-03
CTOL(ABSOLUTE)                             = .1000E-05
FTOL(RELATIVE)                             = .1000E-06
CONSTRAINT NUMBER                             VALUE      THETA
             1                                0.0700      4.01
             2                                0.0800      4.59
INTERVAL OF PHASE CHANGES                  = [-PI,PI]


OBJECTIVE FUNCTION                         =   2.677491D+00
NORM OF THE CONSTRAINT VIOLATIONS          =   1.290319D-08

NO. OF ITERATIONS =    19
    CPU TIME (SECS)                          =  105.980
    OBJECTIVE FUNCTION                       = 2.677
    GAIN                                     =    -0.3399
    NUMBER OF PHASE PERTURBATIONS            =   41
    AVERAGE PHASE PERTURBATION               =     0.00000000
```

I	WEIGHTS	PHASE CHANGES
1	0.264375373671	-0.468410754869
2	0.272106559834	-0.249570927309
3	0.238425718517	-0.373585951615E-01
4	0.314481052305	0.164306088971
5	0.350856533176	0.350742503194
6	0.396897931568	0.513156764071
7	0.450536777329	0.636161908846
8	0.508724360485	0.693381875830
9	0.568254615102	0.640510751069
10	0.626581810598	0.440888273382
11	0.682261419286	0.180113553453
12	0.734854271079	0.114066339518E-01
13	0.784412668475	-0.435999224830E-01
14	0.830863725839	-0.349916940583E-01
15	0.873621668158	-0.288175961065E-02
16	0.911600969048	0.295659727749E-01
17	0.943565035459	0.501396197881E-01
18	0.968565198776	0.544941965626E-01
19	0.986207431884	0.442515323809E-01
20	0.996588198007	0.243283195082E-01
21	1.000000000000	-0.178177334543E-15
22	0.996588198007	-0.243283195082E-01
23	0.986207431884	-0.442515323809E-01
24	0.968565198776	-0.544941955626E-01
25	0.943565035459	-0.501396197381E-01
26	0.911600969048	-0.295659727749E-01
27	0.873621668158	0.288175961065E-02
28	0.830863725839	0.349916940583E-01
29	0.784412668475	0.435999224830E-01
30	0.734854271079	-0.114066339518E-01
31	0.682261419286	-0.180113553453
32	0.626581810598	-0.440888273382
33	0.568254615102	-0.640510751069
34	0.508724360485	-0.693381875830
35	0.450536777328	-0.636161908846
36	0.396897931568	-0.513156764071
37	0.350856533176	-0.350742503194
38	0.314481052305	-0.164306088971
39	0.238425718517	0.373585951615E-01
40	0.272106559834	0.249570927309
41	0.264375373671	0.468410754869

Table 9 Effect of 5 widely spaced constraints on 30 db Taylor pattern. Phase range [-Π,Π] (Fig. 19)

```
BEAM REFERENCE AT CENTRE OF ARRAY
INITIAL PATTERN TAYLOR WEIGHTED
NUMBER OF EQUAL SIDELOBES              =  6
DEPTH OF SIDELOBES                     =  30.0
NUMBER OF ELEMENTS                     =  41
NUMBER OF FITTING POINTS               =  41
DEPTH OF NULL                          =  -90.
NULL INTERVAL                          = [0.400,0.900]
SCALING FACTOR                         = 1.000
EPSI(NULL DEPTH x SCALING FACTOR)= .5927E-03
CTOL(ABSOLUTE)                         = .1000E-05
FTOL(RELATIVE)                         = .1000E-06
CONSTRAINT NUMBER                         VALUE      THETA
            1                             0.4000     23.58
            2                             0.5250     31.67
            3                             0.6500     40.54
            4                             0.7750     50.31
            5                             0.9000     64.16
INTERVAL OF PHASE CHANGES              = [-PI,PI]

EXIT E04VCF - OPTIMAL SOLUTION FOUND.

OBJECTIVE FUNCTION                       =   1.2132170-01
NORM OF THE CONSTRAINT VIOLATIONS =   1.2036490-08

NO. OF ITERATIONS =    38
    CPU TIME (SECS)                        =  525.900
    OBJECTIVE FUNCTION                     = .1213
    GAIN                                   =    -0.0398
    NUMBER OF PHASE PERTURBATIONS          =   41
    AVERAGE PHASE PERTURBATION             =     0.00000000
```

I	WEIGHTS	PHASE CHANGES
1	0.264375373671	0.692330413065E-01
2	0.272106559834	0.202895009211
3	0.288426718517	-0.875112684187E-01
4	0.314481052305	-0.466070249285E-01
5	0.350856583176	-0.166371427073
6	0.396897931668	0.209489964639
7	0.450536777328	-0.383589401507E-01
8	0.508724360485	-0.900332670998E-01
9	0.568254615102	0.336021793505E-01
10	0.626581810598	-0.677896290473E-01
11	0.682261419286	0.122891362484
12	0.734354271079	-0.858541794306E-01
13	0.784412668475	0.362378398464E-01
14	0.830863725839	-0.705272286368E-01
15	0.873621668158	-0.377136036077E-02
16	0.911600969043	0.143176434684
17	0.943565035457	-0.137735778942
18	0.968566198775	0.253295522052E-01
19	0.986207431884	-0.411889637492E-01
20	0.996588138007	0.102325135364
21	1.000000000000	0.326101809204E-10
22	0.996588138007	-0.102325135243
23	0.986207431884	0.411883629452E-01
24	0.968566198775	-0.253295522446E-01
25	0.943563035459	0.137735778319
26	0.911600969048	-0.143176404339
27	0.873621668158	0.377135910495E-02
28	0.830863725839	0.705272284205E-01
29	0.784412668475	-0.362378398627E-01
30	0.734354271079	0.888641794508E-01
31	0.682261419286	-0.122891364740
32	0.626581810598	0.677896293606E-01
33	0.568254615102	-0.336021794185E-01
34	0.508724360485	0.900332689732E-01
35	0.450536777328	0.383389401307E-01
36	0.396897931668	-0.209489964640
37	0.350856583176	0.166371427091
38	0.314481052305	0.466070249417E-01
39	0.288426718517	0.875112684558E-01
40	0.272106559834	-0.202895007867
41	0.264375373671	-0.682330409063E-01

Table 10 Effect of 8 closely spaced constraints on 30 db Taylor pattern. Phase range [-Π,Π] (Fig. 20)

```
BEAM REFERENCE AT CENTRE OF ARRAY
INITIAL PATTERN TAYLOR WEIGHTED
NUMBER OF EQUAL SIDELOBES                =  6
DEPTH OF SIDELOBES                       =  30.0
NUMBER OF ELEMENTS                       =   41
NUMBER OF FITTING POINTS                 =   41
DEPTH OF NULL                            =  -90.
NULL INTERVAL                            = [0.220,0.360]
SCALING FACTOR                           = 1.000
EPSI(NULL DEPTH x SCALING FACTOR)= .5927E-03
CTOL(ABSOLUTE)                           = .1000E-05
FTOL(RELATIVE)                           = .1000E-02
CONSTRAINT NUMBER                            VALUE      THETA
              1                              0.2200     12.71
              2                              0.2400     13.89
              3                              0.2600     15.07
              4                              0.2800     16.26
              5                              0.3000     17.46
              6                              0.3200     18.66
              7                              0.3400     19.83
              8                              0.3600     21.10
INTERVAL OF PHASE CHANGES                = [-PI,PI]

EXIT E04VDF - CURRENT POINT CANNOT BE IMPROVED UPON.

OBJECTIVE FUNCTION                          =   8.935694D-01
NORM OF THE CONSTRAINT VIOLATIONS =   1.532437D-15

NO. OF ITERATIONS =    64
    CPU TIME (SECS)                          =  397.710
    OBJECTIVE FUNCTION                       = .8936
    GAIN                                     =    -0.2281
    NUMBER OF PHASE PERTURBATIONS            =    41
    AVERAGE PHASE PERTURBATION               =     0.00000000
```

I	WEIGHTS	PHASE CHANGES
1	0.264375373671	1.71510277802
2	0.272106559834	0.171956515007E-01
3	0.288426718517	0.366202494836
4	0.314481052305	0.241433479466
5	0.350856583176	-0.148234344926
6	0.396897931668	-0.103902311326
7	0.450536777328	0.184315266762E-01
8	0.508724360485	0.389424677427E-01
9	0.568254615102	0.134648688360E-01
10	0.626581810598	-0.611181412253E-01
11	0.682261419286	-0.111764987877E-01
12	0.734854271079	0.436658318237E-01
13	0.784412668475	0.490156101261E-01
14	0.830863725839	-0.464135535412E-02
15	0.873621668158	-0.439465629398E-01
16	0.911600969048	0.160118748816E-02
17	0.943565035459	0.329653553132E-01
18	0.968566198776	0.372023211178E-02
19	0.986207431884	-0.213953676438E-01
20	0.996538198007	-0.239405317080E-01
21	1.00000000000	-0.152631376268E-10
22	0.996538198007	0.239405318115E-01
23	0.986207431884	0.213953675867E-01
24	0.968566198776	-0.372023197228E-02
25	0.943565035459	-0.329653556752E-01
26	0.911600969048	-0.160118743734E-02
27	0.873621668158	0.439465629512E-01
28	0.830863725839	0.464135536530E-02
29	0.784412668475	-0.490156101400E-01
30	0.734854271079	-0.436658318796E-01
31	0.682261419286	0.111764986415E-01
32	0.626581810598	0.611181412467E-01
33	0.568254615102	-0.134648688269E-01
34	0.508724360485	-0.389424677400E-01
35	0.450536777328	-0.184315266773E-01
36	0.396897931668	0.103902311332
37	0.350856583176	0.148234344823
38	0.314481052305	-0.241433479515
39	0.288426718517	-0.366202494843
40	0.272106559834	-0.171956515138E-01
41	0.264375373671	-1.71510278011

Table 11(a) Effect of 4 constraints on 30 db Taylor pattern Phase range [-Π,Π] (Fig. 21)

```
BEAM REFERENCE AT CENTRE OF ARRAY
INITIAL PATTERN TAYLOR WEIGHTED
NUMBER OF EQUAL SIDELOBES               =  6
DEPTH OF SIDELOBES                      =  30.0
NUMBER OF ELEMENTS                      =   41
NUMBER OF FITTING POINTS                =   41
NUMBER OF CONSTRAINTS                   =    4
DEPTH OF NULL                           =  -90.
NULL INTERVAL                           = [0.220,0.280]
SCALING FACTOR                          = 1.000
EPSI(NULL DEPTH x SCALING FACTOR)= .5927E-03
CTOL(ABSOLUTE)                          = .1000E-05
FTOL(RELATIVE)                          = .1000E-06
CONSTRAINT NUMBER                          VALUE      THETA
            1                             0.2200      12.71
            2                             0.2400      13.89
            3                             0.2600      15.07
            4                             0.2800      16.26
INTERVAL OF PHASE CHANGES               = [-PI,PI]

OBJECTIVE FUNCTION                      =   5.3029650-01
NORM OF THE CONSTRAINT VIOLATIONS =   8.4455330-16

NO. OF ITERATIONS =  350
    CPU TIME (SECS)                        = 1572.720
    OBJECTIVE FUNCTION                     = .5803
    GAIN                                   =   -0.1534
    NUMBER OF PHASE PERTURBATIONS          =   41
    AVERAGE PHASE PERTURBATION             =     0.00000000
```

I	WEIGHTS	PHASE CHANGES
1	0.264375373671	1.42561892443
2	0.272106559834	0.177805834485
3	0.288426718517	0.544411043385E-01
4	0.314481052305	0.352380350719E-01
5	0.350855583176	-0.217133388953E-01
6	0.396897931668	0.160085375483E-02
7	0.450536777328	-0.377313454608E-01
8	0.508724360485	-0.332342720221E-01
9	0.568254615102	0.207684360005E-01
10	0.626581810598	0.180280754370E-02
11	0.682261419286	-0.708933298473E-02
12	0.734854271079	-0.344565083263E-03
13	0.784412658475	-0.102699228359E-01
14	0.830863725839	0.208619030435E-02
15	0.873621668158	0.364915170757E-02
16	0.911600969048	-0.483866915221E-02
17	0.943565035459	0.946326681744E-03
18	0.968566198776	0.250024106192E-02
19	0.956207431884	-0.428633640197E-02
20	0.996583198007	0.257385602219E-02
21	1.000000000000	0.294371351772E-14
22	0.996583198007	-0.257385602220E-02
23	0.936207431884	0.428633640197E-02
24	0.968566198776	-0.250024106192E-02
25	0.943565035459	-0.946326681740E-03
26	0.911600969048	0.483866915221E-02
27	0.873621668158	-0.364715170758E-02
28	0.830863725839	-0.208619030436E-02
29	0.734412663475	0.102699228359E-01
30	0.734854271079	0.344565083265E-03
31	0.682261419286	0.708933298476E-02
32	0.626581810598	-0.180280754370E-02
33	0.568254615102	-0.207684360005E-01
34	0.508724360485	0.332342720221E-01
35	0.450536777328	0.377813454608E-01
36	0.396897931668	-0.160085375473E-02
37	0.350855583176	0.217133388952E-01
38	0.314481052305	-0.352380350720E-01
39	0.288426718517	-0.544411043387E-01
40	0.272106559834	-0.177305834486
41	0.264375373671	-1.42561892448

Table 11(b) Effect of 4 constraints on 30 db Taylor pattern Phase range [0,2Π] (Fig. 22)

```
BEAM REFERENCE AT CENTRE OF ARRAY
INITIAL PATTERN TAYLOR WEIGHTED
NUMBER OF EQUAL SIDELOBES             =   6
DEPTH OF SIDELOBES                    =   30.0
NUMBER OF ELEMENTS                    =    41
NUMBER OF FITTING POINTS              =    41
NUMBER OF CONSTRAINTS                 =     4
DEPTH OF NULL                         =  -90.
NULL INTERVAL                         = [0.220,0.280]
SCALING FACTOR                        = 1.000
EPSI(NULL DEPTH x SCALING FACTOR)= .5927E-03
CTOL(ABSOLUTE)                        = .1000E-05
FTOL(RELATIVE)                        = .1000E-06
CONSTRAINT NUMBER                         VALUE       THETA
            1                             0.2200      12.71
            2                             0.2400      13.89
            3                             0.2600      15.07
            4                             0.2800      16.26
INTERVAL OF PHASE CHANGES             = [0.0,2PI]


OBJECTIVE FUNCTION                        =   1.348706D+00
NORM OF THE CONSTRAINT VIOLATIONS =   1.419232D-03

NO. OF ITERATIONS =    23
    CPU TIME (SECS)                           =    73.930
    OBJECTIVE FUNCTION                        = 1.349
    GAIN                                      =    -0.4411
    NUMBER OF PHASE PERTURBATIONS             =    10
    AVERAGE PHASE PERTURBATION                =     0.34801072
```

I	WEIGHTS	PHASE CHANGES
1	0.264375373671	3.16311495583
2	0.272106559834	0.647509533072
3	0.288426718517	1.56825945656
4	0.314481052305	0.420311448415
5	0.350356583176	0.000000000000E+00
6	0.396897931668	0.000000000000E+00
7	0.450536777323	0.000000000000E+00
8	0.508724360485	0.000000000000E+00
9	0.568254615102	0.000000000000E+00
10	0.626581810598	0.000000000000E+00
11	0.632261419286	0.000000000000E+00
12	0.734854271079	0.000000000000E+00
13	0.784412668475	0.000000000000E+00
14	0.830863725839	0.191025312272
15	0.873621668158	0.928570997621E-01
16	0.911600969048	0.000000000000E+00
17	0.943565035459	0.000000000000E+00
18	0.968566198776	0.000000000000E+00
19	0.986207431884	0.000000000000E+00
20	0.996588198007	0.000000000000E+00
21	1.00000000000	0.000000000000E+00
22	0.996588198007	0.000000000000E+00
23	0.986207431884	0.000000000000E+00
24	0.968566198776	0.000000000000E+00
25	0.943565035459	0.000000000000E+00
26	0.911600969048	0.144310636359
27	0.873621668158	0.569594896799E-01
28	0.830863725839	0.000000000000E+00
29	0.784412668475	0.000000000000E+00
30	0.734854271079	0.000000000000E+00
31	0.632261419286	0.000000000000E+00
32	0.626581810598	0.000000000000E+00
33	0.568254615102	0.000000000000E+00
34	0.508724360485	0.133618278172
35	0.450536777328	0.000000000000E+00
36	0.396897931668	0.000000000000E+00
37	0.350356583176	0.000000000000E+00
38	0.314481052305	0.000000000000E+00
39	0.288426718517	2.01154099199
40	0.272106559834	0.000000000000E+00
41	0.264375373671	0.000000000000E+00